Towards minimum-variance control of ELTs AO systems

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ABSTRACT

Minimum-variance control of adaptive optics (AO) systems relies on a stochastic dynamical model of the perturbation and on models of the components, including loop delays. Resulting LQG controllers have been implemented in SCAO and WFAO both on laboratory benches and on-sky. Their efficiency has been recognized in several modes of operation, e.g. i) on-sky control of TT or low-order modes with vibration mitigation (SPHERE, GPI, CANARY, Raven, GeMS, in H2 formulation at the McMath-Pierce solar telescope) ii) full SCAO mode (CANARY) and MOAO mode (CANARY, Raven) and iv) in general it is advocated to control the low-order modes in laser tomography systems (E-ELT HARMONI LTAO, NFIRAOS). We first point out two examples related to VLT AO controllers to illustrate the need for RTC flexibility. The implementation of LQG control in the framework of the future ELTs raises many questions related both to real-time control computation and associated parameter updates (at a far lower rate), and to the performance that can be reached compared with simpler control strategies. By gathering many lab and on-sky results, we draw the performance trends observed so far. We then outline some promising research directions for control design and implementations for future ELTs AO systems.

Keywords: Adaptive optics control, LQG control, Extremely Large Telescopes

1. TWO LESSONS FROM VLTS

At the time of NAOS first light in the early 2000's, it rapidly appeared that although extraordinary improvement was obtained, the AO system suffered from unforeseen vibrations affecting low-order modes (at least up to the tenth Zernike mode).[44] These vibrations, detected on sky, could not be all filtered out with an optimized integrator in the loop — they could even in some cases be amplified — leading to significant performance degradation: 15% SR loss was reported for some data sets. Since then, one could think that another more adapted controller could have been implemented in the RTC, in order to counteract these nefarious effects. This is not so simple, as it is complicated and costly to upgrade an RTC designed more than 15 years ago.

Drawing on this experience, possible vibration mitigation on tip and tilt modes has been integrated since the early design stage in SAXO,[35] the SPHERE[4] XAO system installed on the VLT. The control scheme features the possible use of a tip/tilt LQG controller combined with an integrator for the high orders modes. It is the only controller on an operational system that adapts itself to perturbation conditions. Impressive results have been obtained, reaching for example more than 90% SR in H band in high flux conditions.[16] The loop sampling frequency was nominally 1.2 kHz, and it has been decided to provide another operational case with 300 Hz sampling frequency. However, the total loop delay in this case appeared to be 1.4 frame, while this LQG controller has been designed for a two-frame delay. This risks to destabilize the control loop, and in order to mitigate that risk, the range of vibrations that can be corrected would have to be limited. In practice, the LQG controller has been limited to correct for turbulence only in this 300 Hz case (i.e. no vibrations are accounted for).[35] Modifying the controller to handle variable loop delays is straightforward (it has been tested on-sky on CANARY in 2014 for the LTAO mode) but requires a slight modification of the Kalman filter and control

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computation in the RTC (see Section 3). This is not so easy, as it is complicated and costly to upgrade an RTC designed recently.

The moral of these stories is that there will always be unforeseen phenomena (due to spurious signals, aging of the components...) or new operating modes (with possibly new components, etc.), and once designed, RTCs are not easily modifiable, so that a natural question that arises is "How far should a controller be flexible to face unanticipated events?"

2. TOWARDS HIGH-PERFORMANCE CONTROL FOR ELTS

Control designers of ELT-sized systems thus include vibration mitigation for low order modes. For example, five modes will be possibly controlled with LQG in TMT NFIRAOS[14] (this is expected to bring significant improvement of sky coverage[50]), the higher orders being corrected with MMSE and pre-conditioning methods.[13,19] For E-ELT HARMONI SCAO and LTAO, LQG control is envisioned for the baseline compensation of tip, tilt and focus modes.[8] It is indeed expected that these types of controllers will also be able to efficiently compensate the very high energy level of the atmospheric turbulence for these modes.[6]

Trends in LQG control performance have been reported for ELT AO systems,[6] showing the potential improvement with respect to integrators. Due to the telescope aperture size, the energy of the low radial orders is much higher than for VLTs, and low radial orders until at least 3 should be considered for high performance control, integrator rejection being not strong enough.[49] A natural question that arises is then "How many low order modes should be accounted for in a high performance ELT AO controller?"

Besides these aspects linked to the atmospheric turbulence, one can also list a number of phenomena that are difficult to fully predict: effect of wind shake and of vibrations on ELT huge structures, leading to disturbance spectra probably even more complex than those observed until now on VLTs, effect of dome turbulence, with non Kolmogorov statistics and non stationary behavior impacting at least low temporal frequency components of the disturbance. Control performance specifications for future ELT AO systems include these phenomena as far as possible. But reality may be ferocious with predictions, leading to potentially envision controller upgrades earlier than expected.

In this regard, keeping some RTC flexibility and scalability to allow for controller evolutions seems reasonable.[5,20,24]

3. MINIMUM VARIANCE AO CONTROL: PERFORMANCE TRENDS

So far, a number of laboratory and on-sky experiments have been conducted with an LQG controller in the AO loop, showing the growing interest of the AO community for these types of control structures, technology evolution having helped considering them as viable. However, minimum variance AO control, the solution of which is an LQG controller,[30] is optimal only with respect to the disturbance model used in the state-space representation. Otherwise, one may hope that the chosen disturbance model is good enough to impact favorably on control performance.

To give a more precise idea of how an LQG AO controller is built, let us take a simple example.[29,30] If ϕ_k is the disturbance and $\phi_k^{\mathrm{cor}} = N u_{k-1}$ is the correction phase generated by the control vector u_{k-1}, with N the influence matrix, the minimum variance control to be applied at time k is obtained as

$$u_k^{\mathrm{mv}} = N^{\dagger} \hat{\phi}_{k+1|k}, \tag{1}$$

where $N^{\dagger}$ is the pseudo-inverse of N and $\hat{\phi}_{k+1|k}$ is the minimum variance estimation of ϕ_{k+1} based on all available measurements until time k. This minimum variance prediction can be obtained as the output of a Kalman filter, provided that a state-space representation of the whole system is available. The difficulty lies therefore in a good prediction of the disturbance, which depends on the chosen model. As proposed in the first lab experiment,[36] let us take a disturbance ϕ simply modelled as a vector-valued auto-regressive process of order 1:

$$\phi_{k+1} = A_1 \phi_k + v_k, \tag{2}$$

where ϕ is the disturbance represented by Zernike coefficients, A_1 is a diagonal matrix with coefficients values depending on the Zernike mode, and v is a white Gaussian noise with known covariance matrix Σ_v. Kolmogorov spatial correlations for ϕ, represented by the covariance matrix Σ_ϕ, are fed into the model by taking for Σ_v the following value

$$\Sigma_v = \Sigma_\phi - A_1 \Sigma_\phi A_1^{\mathrm{t}}. \tag{3}$$

The measurement equation, for a two-frame delay SCAO system, is standardly taken as

$$y_k = D\phi_{k-1} - DNu_{k-2} + w_k, \tag{4}$$

where D is the wavefront sensor matrix and w is the Gaussian white measurement noise. Based on these models, a simple state-space representation is constructed by taking as state vector

$$x = \begin{pmatrix} \phi_k \\ \phi_{k-1} \end{pmatrix}. \tag{5}$$

This leads to the state space model

$$\begin{cases} x_{k+1} = Ax_k + \Gamma v_k \\ y_k = Cx_k - DNu_{k-2} + w_k \end{cases}, \tag{6}$$

with

$$A = \begin{pmatrix} A_1 & 0 \\ I & 0 \end{pmatrix},\ C = (0\ \ D)\ ,\ \Gamma = \begin{pmatrix} I \\ 0 \end{pmatrix} \tag{7}$$

and where I is in each appearance the identity matrix with appropriate dimensions. Any linear modelling of the disturbance can be embedded in a state space representation of the form given in Eq. (6). The corresponding Kalman filter and control computation can then be obtained as

$$\begin{cases} \hat{x}_{k|k} = A\hat{x}_{k-1|k-1} + H_\infty(y_k - CA\hat{x}_{k-1|k-1} + DNu_{k-2}), \\ u_k = N^\dagger C_\phi A \hat{x}_{k|k}, \end{cases} \tag{8}$$

where H_∞ is the asymptotic estimation Kalman gain and C_ϕ is a matrix extracting $\hat{\phi}_{k+1|k}$ from $A\hat{x}_{k|k}$. Accounting for a $(1+\delta)$-frame delay, $0 \leq \delta \leq 1$, instead of 2 simply consists in replacing in the upper line of Eq. (8) u_{k-2} with $\delta u_{k-2} + (1-\delta)u_{k-1}$ and in the lower line A with $\delta A + (1-\delta)I$ (which boils down to $u_k = N^\dagger(\delta\hat{\phi}_{k+1|k} + (1-\delta)\hat{\phi}_{k|k})$).

The disturbance model, and therefore the Kalman filter in Eq. (8), may be used for any set of modes that are to be compensated with an LQG controller, or for the whole disturbance as in.[36,47] It is worth noting that a state-space representation such as Eq. (8) can be used to implement any linear controller,[42] from an integrator to a POLC regulator[12] or an MMSE reconstructor.[17] Therefore, depending on how these equations are coded in the RTC, more or less flexibility will be obtained on the controller structure. This is a crucial point when facing unforeseen disturbances. This flexibility enables to modify the disturbance model structure (i.e. both the disturbance model and the number of modes, or equivalently the size of matrix A, that need to be accounted for) according to future on-sky experiments.

At this stage, an overview of experimental and on-sky results obtained so far is interesting to draw the performance trends of LQG or more generally of observer-based controllers. To do so, we have gathered in Tables 1 and 2 a number of laboratory and on-sky results, with qualitative performance levels as reported by the different authors. The label "Year" corresponds generally to the year of experiment; "Control" details if it is tip/tilt only (TT), and what kind of controller has been used for comparison (e.g., VS integ means a comparison with an integrator); "Model" refers to the structure of perturbation model used to build the Kalman filter (Subspace identification is a case where there is no particular model structure, and where in particular A, C and H_∞ are directly estimated from the data); "Size X" is the size of the state vector; "Param" indicates the parameters needed to tune the controller; "Fs" is the real loop frequency of the hardware; "Perf" indicates wether the performance is consistent with what is expected. The purpose is here to gather many results in order

Teams	Year	Type	Control	Model	Size *X*	Param.	Fs	Perf
Petit & al	2005	SCAO	DM 69+2 vs integ	AR1	278	r0, V	60Hz	=+
		off-axis			600		60Hz	+++
	2006	SCAO		AR1+vibs	278+4	r0, V, 2x2 vibs	60Hz	+++
Hinnen & al	2006	SCAO	DM 37 vs leaky integ	Subspace identification	256	Regularization diag matrix Q	4-20Hz	+++
Costille & al	2009	MCAO 2 DM	DM 52+88 vs integ	AR1, 2L	1240	r0, V, Cn2	10Hz	+++
	2010	LTAO		3 NGS	1240		10Hz	+++
Sivo & al CANARY	2011	SCAO	DM 52+2 vs integ	AR2+vibs	278	r0, V, 2x10 vibs	150Hz	+++
		MOAO	vs APPLY	3 NGS, 2L	860	r0, V, Cn2	150Hz	=+
Agapito & al LBT bench	2011	SCAO	TTM+integ vs integ	AR1+vibs	6	r0, V, 2x1 vib	400Hz	+++
Parisot & al	2011	LTAO 1 DM	DM 52 vs POLC	AR1 4 NGS, 3L	598	r0, V, Cn2	12Hz	+++
Guesalaga & al, GeMS	2013	MCAO	TTM vs integ	AR1+vibs	5	2 vibs	300Hz	+++
Jackson & al, RAVEN	2014	MOAO	DM 97+ MMSE vs spatio-angular LQG	Zonal +Taylor	97	r0, V, Cn2	50Hz	++

Table 1. Laboratory results of LQG or LQG-like controllers. The last column indicates qualitative performance with respect to a reference controller. Related papers: Petit & al,[36,37] Hinnen & al,[25] Costille & al,[9,10] Sivo & al[48] (CANARY bench), Agapito & al[1] (LBT bench), Parisot & al,[34] Guesalaga & al[22] (GeMS bench), Jackson & al[26] (RAVEN bench)

System	Year	Type	Control	Model	Size *X*	Param.	Fs	Perf
@McMath-Pierce	2010	SCAO	TTM	Subspace identification	4	Regularization factor (scalar)	250Hz	+++
CANARY	2012	SCAO	DM52 vs integ	AR2+vibs	556	PSD 2x10 vibs	150Hz	+++
	2013	MOAO	vs APPLY		1960	r0, V, Cn2, 2x10 vibs	150Hz	=+
	2014	LTAO	vs POLC		1960		150Hz	++
RAVEN	2014	MOAO	DM121+MMSE vs spatio-angular LQG	SA LQG AR1	7x121	r0, V, Cn2	100Hz	= =
SPHERE	2014	XAO	TTM	AR2+vibs	44	PSD 2x10 vibs	1.2kHz	+++
GPI	2016	XAO	Tip/tilt/ focus	AR1 Fourier	2304	PSD 2x2 vibs	1kHz	+++
SCExAO	2016	XAO	TTM	AR2+vibs	24	PSD 2x5 vibs	3.5kHz	+++

Table 2. On-sky results of LQG or LQG-like controllers. The last column indicates qualitative performance with respect to a reference controller. Related papers: McMath-Pierce teslescope,[11] CANARY,[46,47] RAVEN,[31] SPHERE,[35] GPI,[41] SCExAO.[32]

to draw some trends: with different systems and operating conditions, is there a significant trend of LQG results to outperform the reference controllers taken by the authors?

It is here time for a small reminder. A couple of years before 2012, SCAO on sky with full LQG control was deemed impracticable, because computationally untractable, certainly too difficult to tune and unable to correct for real turbulence efficiently, because of unadapted priors, not to even mention tomographic LQG... From the results in Table 2, it is clear that with a model rich enough, SCAO LQG or TT LQG (SCAO or XAO) give very good results. A detailed performance analysis would still be needed to better understand what are the error budget terms that are most impacted when using this type of controller.[27] In particular, it would be interesting to compare temporal and aliasing errors when using simplified models for the high-order modes as those used on sky with CANARY[47] with refined dynamical models.

In wide-field, on-sky results in MOAO are of course successful in the sense of running full tomographic LQG controllers on-sky. However, they do not present a clear and systematic improvement compared with the references. The fact that MOAO is open-loop makes it probably more difficult to compensate for model errors, contrarily to the closed-loop case (MCAO, LTAO). It is then natural to ask oneself "Is it possible to derive more efficient disturbance dynamical models?" which concerns wide-field as well as potentially any AO concept. The developments of more efficient models is here a key point, and several solutions have been proposed in order to take advantage of the frozen flow characteristics of the turbulence[39] through explicit (parameterized) or implicit (non parameterized) wind-direction-dependent models.[2, 15, 18, 23, 25, 28, 38, 40] The computational complexity and possible degree of parallelization is then extremely variable.

4. REDUCING THE COMPUTATIONAL BURDEN FOR ELTS IMPLEMENTATION

To reduce the execution time of control computation, besides the usual sequential processing of measurements and commands, parallelization has to be considered altogether for models, controllers and identification schemes. Some highly parallelizable control solutions been proposed[33, 43] but more efforts should be put on combining computational efficiency and control performance. Such developments should be conducted in close relationship with RTC structures, in order to keep all together flexibility, scalability and performance. In that line, the developments of projects like COMPASS,[21] Green Flash[20] or DARC[3] are of particular interest.

Also, depending on the requirements, high-performance controllers need not be implemented on all modes. As an example, taken from,[27] we consider a NAOS-like system in high flux mode with turbulence only (no vibrations). A significant gap in performance can be reached by controlling 9 modes instead of tip and tilt only, the rest being controlled with an integrator, with a drop in residual phase variance of about 20%. Considering what happened with NAOS about vibrations, and the fact that low orders will have much more impact on ELTs,[6] limiting low orders to 3 for an ELT-type AO system seems therefore optimistic. Moreover, control-oriented tools have been recently developed for performance analysis of high-performance controllers,[7, 27] enabling altogether a detailed analysis of a mixed low-order/high-order controller and of disturbance dynamical models efficiency.

With this in mind, RTC flexibility for low orders can be readily obtained by implementing generic controller structures such as those presented in,[27, 42] providing a fully tractable solution when dealing with a few dozen of modes. For higher modes, it is worth mentionning here that, e.g., for MOAO CANARY, the RTC DARC could run at 150 Hz with an LQG controller featuring 2000 components in the state vector[46] which corresponded to about 1000 Zernike modes (4 layers with radial orders 14, 18, 22 and 26) controlled with LQG.[45] This means that with a well optimized control scheme and on future hardware, a few thousand components in the state vector is quite conceivable. And as mentioned before, for even higher dimensions, the massively parallelization capability should be addressed at the design stage for models/controllers/identification schemes, and in close relationship with RTC structures.

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