Synchrotron spectral index in total intensity and polarization

Luke Jew

https://cbass.web.ox.ac.uk/
C-BASS Collaboration

University of Oxford
Richard Grumitt
Jaz Hill-Valler
Luke Jew
Mike Jones
Jamie Leech
Alexander Pollak
Angela Taylor

University of Manchester
Adam Barr
Roke Cepeda-Arroita
Clive Dickinson
Stuart Harper
Paddy Leahy
Mike Peel (Now at Universidade de São Paulo)

Caltech
Tim Pearson
Tony Readhead

South Africa
Moumita Aich (UKZN)
Cynthia Chiang (UKZN/McGill)
Heiko Heiligendorff (UKZN)
Justin Jonas (SKA-SA/Rhodes University)
Sizwe Seranyane (SKA-SA)
Jon Sievers (UKZN/McGill)

KACST
Yaser Hafez
1. C-BASS Data
2. T-T Plots
3. Polarized Spectral Index
1. C-BASS Data

2. T-T Plots

3. Polarized Spectral Index
1. C-BASS Data

See Angela Taylor’s Talk

Nside 64 maps
Beam deconvolved and smoothed to 1 degree
1. C-BASS Data

2. T-T Plots
   - Clustering algorithm
   - Line fitting
   - Results

3. Polarized Spectral Index
2. T-T Plots

Select a region dominated by an emission with a power law frequency-spectrum.
2. T-T Plots
Select a region dominated by an emission with a power law frequency-spectrum.
2. T-T Plots

Select a region dominated by a emission with a power law frequency-spectrum

- Temperature in map A vs. Temperature in map B
- Best fitting straight line
- Slope of line $\Rightarrow$ spectral index
- y-offset $\Rightarrow$ zero levels
2. T-T Plots

Want to measure spectral index between
2. T-T Plots

Want to measure spectral index between

Haslam 408 MHz (Remazeilles et al., 2015)  C-BASS 5 GHz  WMAP K-band 23 GHz (Bennett et al., 2013)
2. T-T Plots

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Haslam 408 MHz  
(Remazeilles et al., 2015)

C-BASS 5 GHz

WMAP K-band 23 GHz  
(Bennett et al., 2013)

• Robust to zero level errors - hence T-T plot method
2. T-T Plots

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  See Richard Grumitt’s Talk

• Use method over all/most of the sky

• There are other emission mechanisms present
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Region selection     Line fitting
2. T-T Plots

Region selection
Use the **mean shift algorithm** to create regions. (with point source mask)

Roughly set the map zero levels and calculate spectral indices

Cluster on:  Sky position
            Haslam/C-BASS spectral index
            C-BASS/WMAP K-band spectral index

Then smooth by a 5 degree Gaussian
2. T-T Plots

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Pixels that are close together with similar spectral properties are grouped.
2. **T-T Plots**

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Automated method of dividing up the entire sky

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**Automated method of dividing up the entire sky**

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See Stuart Harper’s Talk
2. T-T Plots

Line fitting
2. T-T Plots

- Similar S/N across surveys
  - Use the Bayesian method developed by Gull (1989)
- There are pixels that are not dominated by synchrotron emission
  - Use a mixture model to account for outliers
2. T-T Plots

Region 3

Top of NPS

Region 33

Galactic Plane

Region 43

Mid Galactic-Latitudes

\[ \beta = 3.04 \pm 0.00496 (3.04) \]

\[ \beta = 2.74 \pm 0.00635 (2.73) \]

\[ \beta = 2.66 \pm 0.0175 (2.65) \]

\[ \beta = 2.91 \pm 0.00802 (2.91) \]

\[ \beta = 2.11 \pm 0.00332 (2.11) \]

\[ \beta = 2.31 \pm 0.0206 (2.31) \]
2. T-T Plots

C-BASS / Haslam

\[ \langle \beta \rangle = 2.7 \]

WMAP K-band / C-BASS

\[ \langle \beta \rangle = 2.2 \]
2. T-T Plots

The diagram shows a scatter plot with two axes: 

- \( \beta \) WMAP K-band/C-BASS on the y-axis
- \( \beta \) C-BASS/Haslam on the x-axis

The plot includes points categorized by different ranges of \( b \) angles:
- \( b < 30^\circ \) represented by light green triangles
- \( 30^\circ < b < 60^\circ \) represented by orange squares
- \( 60^\circ < b < 90^\circ \) represented by dark purple circles

A red diagonal line represents the trend for different \( b \) ranges.
2. T-T Plots

![Graph showing T-T Plots with spectral curvature on the y-axis and distance from the Galactic Plane on the x-axis.](image-url)
1. C-BASS Data
2. T-T Plots
3. Polarized Spectral Index
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Want to measure polarized spectral index between
3. Polarized Spectral Index

Want to measure polarized spectral index between

C-BASS 5 GHz

Planck 30 GHz

(Planck Collaboration, 2018)
Want to measure polarized spectral index between **C-BASS 5 GHz** and **Planck 30 GHz** (Planck Collaboration, 2018)

**Power-law frequency-spectrum, pixel-by-pixel**

\[ P_i(n) = A_0(n) \left( \frac{\nu_i}{\nu_0} \right)^{-\beta(n)} , \]
Power-law frequency-spectrum

\[ P_i(n) = A_0(n) \left( \frac{\nu_i}{\nu_0} \right)^{-\beta(n)}, \]

Polarized intensity is a Rician random variable (if \( \sigma_Q \approx \sigma_U \))

\[
p(P_1, P_2 | A_0, \beta, \sigma_{P1}, \sigma_{P2}) = \frac{P_1}{\sigma_{P1}^2} e^{-\frac{p_1^2+(A_0(\nu_1/\nu_0)^{-\beta})^2}{2\sigma_{P1}^2}} I_0 \left( \frac{P_1 A_0(\nu_1/\nu_0)^{-\beta}}{\sigma_{P1}^2} \right) \times \]
\[
\frac{P_2}{\sigma_{P2}^2} e^{-\frac{p_2^2+(A_0(\nu_2/\nu_0)^{-\beta})^2}{2\sigma_{P2}^2}} I_0 \left( \frac{P_2 A_0(\nu_2/\nu_0)^{-\beta}}{\sigma_{P2}^2} \right).
\]

Prior

\[ p(A_0)p(\beta) \propto A_0 |\beta| \]

Jeffreys prior for the amplitude of a Rician random variable, and assigning

\[ p \left( (\nu_i/\nu_0)^{-\beta} \right) \propto \text{constant} \]

For the gory details see https://ora.ox.ac.uk/objects/uuid:31f0227a-84be-421a-ae46-eabe9f422767
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Power-law frequency-spectrum

\[ P_i(n) = A_0(n) \left( \frac{v_i}{v_0} \right)^{-\beta(n)} \]

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Not noise biased - even in low S/N
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$

$\beta|_{b>20^\circ} = 3.073$
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, normalised deviation from mean

\[
\hat{\beta}_{b>20^\circ} = 3.073
\]
3. Polarized Spectral Index
C-BASS/Planck 30 GHz

$$\hat{\beta}_{|b>20^\circ} = 3.073$$
Downgrading to lower Nside, the uncertainty on the weighted average of pixels drops quickly.

All on the same colour scale.
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$

$\hat{\beta}_{|b>20^\circ} = 3.073$
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$

$\hat{\beta}_{|b>20^\circ} = 3.073$
3. Polarized Spectral Index

C-BASS/Planck 30 GHz, $\beta$

$\hat{\beta}|_{b>20^\circ} = 3.073$

Nside 4
3. Polarized Spectral Index
C-BASS/Planck 30 GHz

<table>
<thead>
<tr>
<th>Latitude range</th>
<th>$\beta$</th>
<th>$\sigma_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole sky</td>
<td>2.6770</td>
<td>0.0004</td>
</tr>
<tr>
<td>$0^\circ&lt;b&lt;10^\circ$</td>
<td>2.5640</td>
<td>0.0005</td>
</tr>
<tr>
<td>$10^\circ&lt;b&lt;20^\circ$</td>
<td>2.902</td>
<td>0.001</td>
</tr>
<tr>
<td>$20^\circ&lt;b&lt;30^\circ$</td>
<td>3.035</td>
<td>0.002</td>
</tr>
<tr>
<td>$30^\circ&lt;b&lt;50^\circ$</td>
<td>3.102</td>
<td>0.002</td>
</tr>
<tr>
<td>$50^\circ&lt;b&lt;75^\circ$</td>
<td>3.087</td>
<td>0.003</td>
</tr>
<tr>
<td>$75^\circ&lt;b&lt;90^\circ$</td>
<td>3.156</td>
<td>0.009</td>
</tr>
<tr>
<td>$b&gt;20^\circ$</td>
<td>3.073</td>
<td>0.001</td>
</tr>
</tbody>
</table>

S-PASS: $\beta=3.22 \pm 0.08$ (2.3—33 GHz)

(Krachmalnicoff et al., 2018)
Angular power spectrum of polarized spectral index, $b>25^\circ$

- Miville-Deschênes et al. (2008) Used by PySM
- Krachmalnicoff et al. (2018)

$D_\ell^{\beta\beta}$ for $\beta$ C-BASS/Planck 30 GHz
Compare I and P

\[ \beta \text{ Planck 30 GHz/C-BASS (Polarization)} \]

\[ \beta \text{ C-BASS/Haslam (Total intensity)} \]

- \( b < 30^\circ \)
- \( 30^\circ < b < 60^\circ \)
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