Population synthesis models
From stellar evolution models to synthetic populations in the Milky Way

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Lecture 2 scheme

Lecture goals:

- building isochrones
  - what information is in the tabulated isochrones?
  - conversion to various photometric systems
  - extinction

- simulating star clusters
  - sampling the initial mass function
  - binaries
What is an isochrone?

Evolutionary track: locus of star of given initial mass $M_i$ as a function of age $t$

Isochrone: locus of stars of a given age $t$ as a function of initial mass $M_i$

Isochrones are built from dense grids of evolutionary tracks $M_i$ via interpolation, for every evolutionary property $a$ (like $L$, $T_{\text{eff}}$, $M_{\text{core}}$, $\{X_i\}$)

$$a(t)|_{M_i=\text{cte.}} \rightarrow a(M_i)|_{t=\text{cte.}} \quad (1)$$

The only technical problem: given the wide changes in evolutionary timescales with $M_i$, unless grid of tracks is extremely dense ($\Delta M_i \rightarrow 0$), interpolation has to be performed between points of equivalent evolutionary status in adjacent tracks.
Graphical representation

Building isochrone sections:

- just a matter of creating many pairs of equivalent points between the tracks
- a few of these pairs are really equivalent
- others pairs are interpolated in age along each track section
- finally, interpolate for the right age, between equivalent pairs.
Graphical representation

\[ M_i \times t \times \text{phase} \]
- post-main sequence part of isochrone has very short range in \( M_i \)
- this is why it *looks like* a single track
- but resemblance is not always true, see e.g. 1.75 \( M_\odot \) interval

\[ M_i \times t \times \log L/L_\odot \] (Girardi et al. 2013)
Example: PARSEC isochrones

Isochrones and star clusters

How isochrones are made

Example: PARSEC isochrones

Z=0.008 Y=0.263 scaled-solar

log L/L☉ vs log T eff

IMS + HMS

pre-MS

12, 5.6, 4, 2.8, 2.2, 1.8 M☉

ZAMS to TP-AGB or C-ignition

LMS + VLMS

pre-MS

175, 14, 1, 0.6, 0.3, 0.1 M☉

ZAMS to TRGB or 10⁸yr

LMS

ZAHB to TP-AGB or post-HHB

0.484, 0.5, 0.53, 0.6, 1.0, 1.75 M☉

Log(L/L☉) vs Log(T eff)

log(t/yr) from 6.0 to 10.2, Δlogt=0.1 [M/H]=0.0

derived isochrones at equally-spaced intervals in log t

a complete set of tracks for 1 metallicity
Isochrones and star clusters

How isochrones are made

Example: PARSEC isochrones

CMD 2.5 input form

A web interface dealing with stellar isochrones and their derivatives

New! (13mar13) Several photometric systems added.
(07feb13) Format of isochrones in WFC3 wide systems changed.
(26jan13) PARSEC isochrones v1.1 released! with revised diffusion+overshooting at low masses (see Bressan et al., 2012), increased range of ages at low metallicities, and finer details in the pre-MS phase.

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Evolutionary tracks

New: PARSEC isochrones from Bressan et al. (2012), with scaled-solar composition and following the $Y=0.2485+1.78Z$ relation. The solar metal content is $Z_{\text{sun}}=0.0152$. They include the pre-main sequence phase.

Note: We are still extending these isochrones. New versions will be released every few weeks.

Warning: for the moment, PARSEC isochrones do not include the TP-AGB. We are working on this.

PARSEC version 1.1: available for $0.0001\leq Z\leq 0.03$ (2.2Z/M/H=0.5), in the range $0.1\leq M/M_\odot <12$. With revised diffusion+overshooting in low-mass stars, and improvements in interpolation scheme.

PARSEC version 1.0: available for $0.0001\leq Z\leq 0.07$ (-1.5Z/M/H=0.6), in the range $0.1\leq M/M_\odot <12$.

Mass-loss on RGB using the Reimers formula with $\eta_{\text{Reimers}}=0.2$

Warning: mass loss works fine as long as $\eta_{\text{Reimers}}<0.5$. Check the results for higher values.

Previous sets: The following isochrones are available for all $0.0001\leq Z\leq 0.03$, and ages from 0 to 17 Gyr, in the range $0.15\leq M/M_\odot <100$.

Marigo et al. (2008) with the Girardi et al. (2010) Case A correction for low-mass, low-metallicity AGB tracks

as above but for Case B

Marigo et al. (2008): Girardi et al. (2000) up to early-AGB + detailed TP-AGB from Marigo & Girardi (2007) (for $M_7M_\odot$) + Bertelli et al. (1994) (for $M_7M_\odot$) + additional $Z=0.0001$ and $Z=0.001$ tracks.

Basic set of Girardi et al. (2002): Girardi et al. (2000) + simplified TP-AGB (for $M_7M_\odot$) + Bertelli et al. (1994) (for $M_7M_\odot$) + additional $Z=0.0001$ and $Z=0.001$ tracks.

Photometric system

http://stev.oapd.inaf.it/cmd

allows to specify the set of tracks, ages, metallicities, photometric system, extinction, ...
Isochrones and star clusters

How isochrones are made

Example: PARSEC isochrones

output table
How isochrones are “coloured”

Basic reference: Girardi et al 2002. All we need are:

- **spectral energy distributions (SEDs)** for all stars in our isochrone database. Most relevant variables for the SED are the effective temperature $T_{\text{eff}}$, surface gravity $g$, and surface chemical composition $\{X_i\}$:

  $$F_\lambda(T_{\text{eff}}, g, \{X_i\})$$

- **filter (+telescope+detector) transmission curves**, $S_\lambda$

- **zeropoints**, or the reference spectra and magnitudes that define them.
The flux as it arrives at the Earth, $f_\lambda$, is related to the flux at the stellar surface, $F_\lambda$, by

$$f_\lambda = 10^{-0.4A_\lambda} (R/d)^2 F_\lambda,$$

where

- $R$ is the stellar radius,
- $d$ is its distance, and
- $A_\lambda$ is the extinction in magnitudes at the wavelength $\lambda$.

The apparent magnitude $m_{S_\lambda}$, in a given pass-band with transmission curve $S_\lambda$ comprised in the interval $[\lambda_1, \lambda_2]$, is given by

$$m_{S_\lambda} = -2.5 \log \left( \frac{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda S_\lambda d\lambda}{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda^0 S_\lambda d\lambda} \right) + m_{S_\lambda}^0$$

Note: this equation integrates photon counts (as modern CCDs and IR-arrays do), not photon energy (as used to be for old photomultipliers).
How isochrones are “coloured”

We want to derive the absolute magnitudes $M_{S\lambda}$ for each star of known $(T_{\text{eff}}, g, [M/H])$ – and hence known $F_{\lambda}$. This can be obtained by means of Eq. (4) once a distance of $d = 10$ pc is assumed, i.e.

$$M_{S\lambda} = -2.5 \log \left[ \left( \frac{R}{10 \, \text{pc}} \right)^2 \frac{\int_{\lambda_1}^{\lambda_2} \lambda F_{\lambda} 10^{-0.4A_{\lambda}} S_{\lambda} \, d\lambda}{\int_{\lambda_1}^{\lambda_2} \lambda f_0 S_{\lambda} \, d\lambda} \right] + m_0^{S\lambda} \quad (5)$$

and once the stellar radius $R$ is known.

Since the quantities $(T_{\text{eff}}, g, [M/H])$ are not enough to specify $R$ (in this case we need to have also the stellar mass $M$), we should first eliminate $R$ from our equations. This is possible if we deal with the bolometric corrections,

$$BC_{S\lambda} = M_{\text{bol}} - M_{S\lambda} \quad (6)$$
How isochrones are “coloured”

From the definition of bolometric magnitude, we have (see also Bessell et al. 1998 for a similar approach):

$$M_{\text{bol}} = M_{\text{bol}, \odot} - 2.5 \log(L/L_\odot)$$
$$= M_{\text{bol}, \odot} - 2.5 \log(4\pi R^2 F_{\text{bol}}/L_\odot),$$

where $F_{\text{bol}} = \int_0^\infty F_\lambda \, d\lambda = \sigma T_{\text{eff}}^4$ is the total emerging flux at the stellar surface.

Substituting Eqs. (5) and (7) into Eq. (6), we get

$$BC_{S\lambda} = M_{\text{bol}, \odot} - 2.5 \log \left[ 4\pi (10 \text{ pc})^2 F_{\text{bol}}/L_\odot \right]$$
$$+ 2.5 \log \left( \frac{\int_{\lambda_1}^{\lambda_2} \lambda F_\lambda 10^{-0.4A_\lambda S_\lambda d\lambda}}{\int_{\lambda_1}^{\lambda_2} \lambda f_\lambda^0 S_\lambda d\lambda} \right) - m_{S\lambda}^0$$

that, as expected, depends only the spectral shape ($F_\lambda 10^{-0.4A_\lambda} / F_{\text{bol}}$), and on basic astrophysical constants. We adopt $M_{\text{bol}, \odot} = 4.77$, and $L_\odot = 3.844 \times 10^{33}$ erg s$^{-1}$ (Bahcall et al. 1995).
How isochrones are “coloured”

By means of Eq. (8), we tabulate $BC_{S\lambda}$ for all spectra in our input library, and for several different photometric systems. The $BC_{S\lambda}$ can be then derived for any intermediate $(T_{\text{eff}}, g, [M/H])$ value, by interpolation in the existing grid.

If grid is dense enough, may use simple linear interpolations with log $T_{\text{eff}}$, log $g$, and [M/H] as the independent variables.

Then, to attribute absolute magnitudes to stars of given $(T_{\text{eff}}, L)$ along an isochrone, we simply compute $M_{\text{bol}}$ with Eq. (7), and hence

$$M_{S\lambda} = M_{\text{bol}} - BC_{S\lambda}.$$
Isochrones and star clusters

Zeropoints

$f^0_\lambda$ is a reference spectrum that produces a known apparent mag $m^0_{S\lambda}$.

**VEGAmag systems** make use of Vega ($\alpha$ Lyr) as the primary calibrating star.

E.g.

- Johnson-Cousins-Glass $UBVRIJHKLMN$ can be recovered reasonably well by assuming Vega has $V = 0.03$ mag and all colours = 0.
- HST stellar observations are largely expressed in Vegamags
- all post-2MASS infrared systems are Vegamags, with just tiny corrections derived from post-survey calibration issues.

Calibrated empirical spectra of Vega:

- Hayes 1985, covering $\lambda$ from 3300 to 10500 Å,
- extended up to 1150 Å when complemented with IUE spectra (Bohlin et al. 1990).

Alternatively, synthetic spectra from Castelli & Kurucz (1994), with $T_{\text{eff}} = 9550$ K, $\log g = 3.95$, $[M/H] = -0.5$, and $\xi = 2$ km s$^{-1}$.

$F_\lambda$ has to be scaled by the geometric dilution factor

$$(R/d)^2 = (0.5 \theta_d/206264.81)^2,$$

where $\theta_d$ is the observed Vega’s angular diameter (in arcsec) corrected by limb darkening.


Zeropoints

**ABmag systems** assume that a reference spectrum of constant flux density per unit frequency

\[
f_{\text{AB},\nu}^0 = 3.631 \times 10^{-20} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}
\]

(11)

will have AB magnitudes \( m_{\text{AB},\nu}^0 = 0 \) at all frequencies \( \nu \).

Following Oke (1964), monochromatic AB magnitudes are defined by

\[
m_{\text{AB},\nu} = -2.5 \log f_{\nu} - 48.60
\]

(12)

ABmag systems just extend this definition, replacing the monochromatic flux \( f_{\nu} \) with the photon counts over each pass-band \( S_{\lambda} \) obtained from the star, compared to the photon counts that one would get by observing \( f_{\text{AB},\nu}^0 \):

\[
m_{\text{AB},S_{\lambda}} = -2.5 \log \left[ \frac{\int_{\lambda_1}^{\lambda_2} (\lambda/hc)f_{\lambda}S_{\lambda}d\lambda}{\int_{\lambda_1}^{\lambda_2} (\lambda/hc)f_{\text{AB},\lambda}^0S_{\lambda}d\lambda} \right],
\]

(13)

where \( f_{\text{AB},\lambda}^0 = f_{\text{AB},\nu}^0 c/\lambda^2 \).

ABmags are becoming popular with new wide-area optical surveys: SDSS, CFHT/Megacam, DES, PanSTARSS-1, ...
Zeropoints

“Standard stars” systems use a set of a few reference stars, e.g. bright blue stars different from Vega, white dwarfs, or metal-poor subdwarfs.

Good examples are the Vilnius (Straizys & Zdanavicius 1965) and Thuan-Gunn (Thuan & Gunn 1976) systems.

A good set of standards is provided by the four metal-poor subdwarfs BD +17°4708, BD+26°2606, HD 19445 and HD 84937, which are widely-used spectrophometric secondary standards (Oke & Gunn 1983), as well as standards for several photometric systems.
Some useful grids of SEDs

- MARCS
- PHOENIX
- COMARCS for C stars (Aringer et al. 2008)
Extinction coefficients

Basic references:

- Grebel & Roberts 1995, Girardi et al. 2010 (extinction coefficients, heterochromatic extinction)

Relative interstellar extinction coefficients:

$$A_{S_\lambda}/A_V = [BC_{S_\lambda}(0) - BC_{S_\lambda}(A_V)]/A_V$$

where $BC_{S_\lambda}(A_V)$ are computed with Eq. 8 and $A_V > 0$, and the 0 stands for the $A_\lambda = 0$ case.
Extinction coefficients

Note: $A_{S\lambda}/A_V$ slowly varies with $T_{\text{eff}}$, $g$ and $A_V$ (Forbes effect)!

Girardi et al. 2010:

$$A_{S\lambda}/A_V \propto T_{\text{eff}}$$

$$A_{S\lambda}/A_V \propto A_V$$

Effect is minor, but for very broad filters, for the UV, and for very cool giants in the optical.
The initial mass function

Gives a prescription for the numbers of stars populating each isochrone section $[M_i, M_i + dM_i]$, e.g. for Salpeter-like IMFs:

$$
\phi_{M_i} = \frac{dN}{dM_i} = \begin{cases} 
C M_i^{-(1+x)} & \text{for } M_{i\text{inf}} < M_i < M_{i\text{sup}} \\
0 & \text{for } M_i < M_{i\text{inf}} \text{ and } M_i > M_{i\text{sup}} 
\end{cases}
$$

where a possible choice for $C$ is the one that gives

$$
\int_0^\infty \phi_{M_i} dM_i = 1 \text{ star}
$$

(16)

giving

$$
C = \frac{x}{M_{i\text{inf}}^{1-x} - M_{i\text{sup}}^{1-x}}.
$$

(17)

Another possible choice (see lecture 3) of normalization is

$$
\int_0^\infty \phi_{M_i} M_i dM_i = 1 \, M_\odot
$$

(18)
The initial mass function

How do we populate the IMF? See e.g. Girardi & Bica 1993 or Santos & Frogel 1997 for simple examples. See also Numerical Recipes, Monte Carlo chapter.

\( \phi(M_i) \) is a PDF for the mass \( M_i \), transform it into a new PDF for the variable \( N \) with

\[
g(N) = 1, \quad 0 \leq N \leq 1. \tag{19}
\]

where \( N \) can easily be produced by any random-number generator. With

\[
|\phi(M_i)dM_i| = |g(N)dN|, \tag{20}
\]

we have that

\[
M_i = [M_{i,\text{inf}}(1 - N) + M_{i,\text{sup}}N]^{-1/x}. \tag{21}
\]

which can easily be located inside the isochrone table.
The initial mass function

But you can simply do it graphically (or numerically):

1. Generate random numbers here.

2. Locate the random numbers in the mass coordinate here.
Populating isochrones

The result: stars locate where the $M_i$ more rapidly varies.
Apparent and detached binaries

**Apparent binaries:** just take 2 stars at random and add their light, in quantities proportional to local stellar density (crowding)

**Detached binaries:** same as before, but
- proportional to number of stars, not density
- mass ratio is not random, needs either theoretical or empirical formula for it

In both cases, main effects on the CMDs are:
- binary sequence parallel to MS, up to 0.7 mag brighter
- some spread towards bluer/brighter part of red clump and RGB
Apparent and detached binaries

Hurley & Tout 1998:

\[ \text{ Isochrones and star clusters } \quad \text{Binaries} \]

\[ \text{Apparent and detached binaries} \]

\[ \text{Hurley & Tout 1998:} \]

\[ \text{Population synthesis models} \]

\[ \text{GAIA/ITN School, Tenerife, Sep. 2013} \]
Interacting binaries

- stars close enough to interfere with each others’ evolution: wide range of possible fates, depending on initial parameters
- responsible for lots of exotica: blue stragglers, X-ray binaries, symbiotic stars, cataclismic variables, novae, SN Ia, ...
- evidence of being more important in dense environments (e.g. core-collapsed globular clusters), and in massive stars

Example: BSE evolution of a $2.9 \ M_\odot + 0.9 \ M_\odot$ binary with $P = 8$ days, $e = 0.7$. 
Interacting binaries

- assuming initial distribution of mass ratios, semi-major axis, eccentricities, ..., (plus many other assumptions)
- binaries populations can be modeled and even inserted into N-body models

Hurley et al. 2001
Next lecture

Adding star clusters to make galaxies.

Homework: Take an isochrone from any database, and generate a synthetic HR diagram with 1000 stars following the Salpeter IMF from it. Do it directly with a plotting-capable code (supermongo, IDL, python). Then try to do the same for a given initial mass, say for a cluster formed with $10^5 M^\odot$. 

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