Stellar Abundances

I. Basic principles of stellar nucleosynthesis
II. Basic ingredients
III. Deciphering abundances
IV. A case study: LiBeB

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GREAT-ITN School, IAC - 2013
Basic ingredients

- Stellar atmospheres and line formation
- Characteristics of star → stellar parameters
- From lines to abundances
- Lines (atomic and/or molecular)
- Model atmospheres
- Available tools
Abundance Scales (and nomenclature)

Mass fractions

\[ X = \text{H}; \ Y = \text{He}, \ Z = \text{all other elements (}=\text{metals}) \]

\[ X + Y + Z = 1 \]

EX: cf. 1st lecture

\[ X_{\text{sun}} = 0.7381, \ Y_{\text{sun}} = 0.2485, \ Z_{\text{sun}} = 0.0134 \]

(Asplund et al. 2009)

The 12 scale

\[ \log \varepsilon(X) = \log \left( \frac{n_X}{n_H} \right) + 12 \left( \log \varepsilon(H) \equiv 12 \right) \]

EX: \( \log \varepsilon(O)_{\text{sun}} \approx 8.7 \) dex \hspace{1em} (O is \( \approx 2000\times \) less abundant than \( H_{\text{sun}} \))

The [] scale

\[ \left[ \frac{X}{H} \right] = \log \left( \frac{n_X}{n_H} \right) - \log \left( \frac{n_X}{n_H} \right)_{\text{sun}} \]

EX: \[ \left[ \frac{\text{Fe}}{H} \right] = -2 \rightarrow \text{star has } 1/100 \text{ less iron than the Sun} \]

\[ \left[ \frac{\text{Fe}}{H} \right] = 0.5 \rightarrow \text{star has } 3.16\times \text{ more iron than the Sun} \]
### Table 1  Element abundances in the present-day solar photosphere. Also given are the corresponding values for CI carbonaceous chondrites (Lodders, Palme & Gail 2009). Indirect photospheric estimates have been used for the noble gases (Section 3.9)

<table>
<thead>
<tr>
<th>Z</th>
<th>Element</th>
<th>Photosphere</th>
<th>Meteorites</th>
<th>Z</th>
<th>Element</th>
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<td>1.75 ± 0.08</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>C</td>
<td>8.43 ± 0.05</td>
<td>7.39 ± 0.04</td>
<td>49</td>
<td>In</td>
<td>0.80 ± 0.20</td>
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<td>N</td>
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<td></td>
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<td>0.90 ± 0.20</td>
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<td>Sr</td>
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<td>2.88 ± 0.03</td>
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<td>Pb</td>
<td>1.75 ± 0.10</td>
<td>2.04 ± 0.03</td>
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<tr>
<td>39</td>
<td>Y</td>
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<td>Bi</td>
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<td>2.53 ± 0.04</td>
<td>90</td>
<td>Tb</td>
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<td>0.06 ± 0.03</td>
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<td>Mo</td>
<td>1.88 ± 0.08</td>
<td>1.94 ± 0.04</td>
<td></td>
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</table>
How do you extract elemental abundances from these lines?

- Star ID (spectral type or photometric classification)
- Atmospheric properties → line formation
  - Effective temperature
  - Surface gravity
  - “Metallicity”
• Stellar atmospheres and line formation
  • Characteristics of star $\rightarrow$ stellar parameters
  • From lines to abundances
  • Lines (atomic and/or molecular)
  • Model atmospheres
  • Available tools
What information does the observed flux carry?

- Absorption-line spectrum
- Lines occur because of atomic absorption → Estimation of elements in the photosphere
- Emergent flux carries information about physical conditions in photosphere
Stellar photosphere: a definition

Internal structure:
- inner core
- radiative zone
- convection zone

Subsurface flows

Photosphere

Chromosphere

Corona

Transition from interior to ISM

Layer from which we receive photons

Source: NASA
Structure of the Sun

- **R = 700,000 km**
- Photosphere ≈ 500 km
  ≈ 1% of R
- **ρ = 130 g/cm³**
- **T = 15 million K**
- **P = 200 billion Atm**
- **T = 5800 K**
- **P = 0.09 Atm**
- **ρ = 2 \times 10^{-7} g/cm³**

*Source: Heiter*
A cool, thin gas seen in front of a hot source produces absorption lines: in the continuum region, $\tau$ is low and we see primarily the background source. At the wavelengths of spectral lines, $\tau$ is large and we see the intensity characteristic of the temperature of the cool gas. Since this is lower than central source, these appear as absorption features.

\[ \tau_v = \int_0^L k_v \rho dx \]
Spectral line formation & profile

The formation of absorption lines can be qualitatively understood by studying how $S_\nu$ changes with depth.

\[ W_{\lambda} \propto \frac{d(\ln S_{\nu})}{d\tau_{\nu}} \]

1. Atmospheric structure

2. Absorption lines: negative T gradient \( \rightarrow \) source function \( \downarrow \) outwards

3. Transfer equation: solution provides detailed line profile

Source: Gray
Ionization state & energy level

What we want:

1. Enough atoms in the right ionization state (LTE assumption)

   Saha’s equation

   \[ \frac{n_1}{n_0} P_e = \frac{(2\pi m_e)^{3/2}(kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-\frac{1}{kT}} \]

2. Enough atoms or ions excited in the right energy level (LTE assumption)

   Boltzmann’s statistics

   \[ \frac{N_b}{N_a} = \left( \frac{g_b}{g_a} \right) e^{-\frac{(E_b - E_a)}{kT}} \]

Combination of both →
Line broadenings

3 main components

**Natural width** (Lorentzian profile, very narrow)
- due to Heisenberg uncertainty principle $\Delta E \cdot \Delta t = h/2\pi$

**Thermal (Doppler) width** (Gaussian, Maxwell-Boltzmann distribution)
- thermal motions of the atoms, randomly distributed shifts $\rightarrow$ centre

**Pressure** (collisional) width (Lorentzian profile)
- collisions between particles (energy levels change) $\rightarrow$ wings
More broadenings, splits and shifts

Zeeman
In strong magnetic fields atoms can align in quantum ways causing slight separations in the energies of atoms in the same excitation levels.

This “splits” the lines into multiple components. The stronger the field, the greater the splitting.

Hyperfine
Interaction between the nuclear magnetic dipole with the magnetic field of the electron can perturb the energy levels of an atom

Energy level dependent → different for each line

Stark
Perturbation by electric fields
Some examples: Hydrogen lines

Balmer transitions (from the n=2 excited level): strongest in A stars because that is where N (HI, n=2) is highest
Examples: Helium lines

Second most abundant element
Detectable only in very hot stars, O- and B-types (even Hell in hottest O-stars)

Examples: metal lines

Line strength depends on level population of the atoms.

Strongest when temperature is low (lower ionization stages are populated)

Lines become stronger as $T$ decreases

Dominate in F, G, K stars

Examples: molecular bands

Form in very cool stars (M-, L-, T-types)

Can vibrate and rotate, besides having energy levels

Flux significantly reduced in bands

Electron transitions:
Visible+UV lines

Vibrational transitions:
Infrared lines

Rotational transitions:
Radio-wave lines

M-stars: TiO
L- / T-stars: CO, H₂O and CH₄

Source: Kirckpatrick et al, 1999, 2000
Examples: dominant features
• Stellar atmospheres and line formation

• **Characteristics of star ➔ stellar parameters**
  
  • From lines to abundances
  
  • Lines (atomic and/or molecular)
  
  • Model atmospheres
  
  • Available tools
What are the stellar parameters?

Characteristics of a star:

1. $M$, $L$, $X$, $Y$, $Z$, $R$, $v_{rot}$, $t$, ... \textit{i.e., the stellar-structure view}

2. $F_{\nu}$, $T_{\text{eff}}$, $\log g$, $[X/H]$, $v_{rot} \sin i$, $\log (G M / R^2)$ ... \textit{i.e., the stellar-atmosphere view}

1 \rightarrow \text{more fundamental}

2 \rightarrow \text{more empirical}

\textbf{Sun}

$M = 2 \times 10^{33} g = M_{\text{sun}}$

$R = 7 \times 10^{10} \text{cm} = R_{\text{sun}}$

$L = 4 \times 10^{33} \text{erg/s} = L_{\text{sun}}$

$R_{\text{phot}} \sim 200 \text{ km} < 10^{-3} R_{\text{sun}}$

$N_{\text{phot}} \sim 1015 \text{cm}^{-3}$

$T_{\text{eff}} \sim 5780 \text{K}$

\textbf{O star}

$M \sim 50 M_{\text{sun}}$

$R \sim 20 R_{\text{sun}}$

$L \sim 10^6 L_{\text{sun}} (\alpha M^3)$

$R_{\text{phot}} \sim 0.1 R_{\text{sun}}$

$N_{\text{phot}} \sim 1014 \text{cm}^{-3}$

$T_{\text{eff}} \sim 40000 \text{K}$

\textit{Source: Korn}
Fundamental stellar parameters: how to

- $T_{\text{eff}}$ via $F_{\text{Bol}}$ and $\Theta$
  
  For $\Theta$, one uses interferometry and model-atmosphere theory (limb darkening)

- $\log g$
  
  Newton’s law, needs $M$ and $R$
  
  One needs $\pi$ (parallax) and $\Theta$
  
  Gaia is the key $\pi$-mission (soon to be launched)

- $M$
  
  Eclipsing binaries (very limited)
  
  Must be inferred from stellar evolution

- $[m/H]$ via meteorites for the Sun

  Caveat: lack of important volatile elements like CNO and noble gases
Most relevant empirical stellar parameters: how to

Effective temperature $T_{\text{eff}} = T_{\text{BB}}$ with same L and R as the real star

$L = 4\pi R^2 \sigma T_{\text{eff}}^4$

Surface gravity $\log g$ usually expressed in cgs units and as $\log_{10}$

$g = GM/R^2$

Metallicity $Z$ or $[\text{Fe/H}]$

$Z = \frac{\text{mass (elements heavier than He)}}{\text{total mass (unit volume)}} \approx 0.018 \quad \text{Sun}$

$[\text{Fe/H}] = \log \frac{N(\text{Fe})}{N(\text{H})} - \log \frac{N(\text{Fe})}{N(\text{H})}_{\text{Sun}}$

$[\alpha/\text{Fe}] = \log \frac{N(\alpha)}{N(\text{Fe})} - \log \frac{N(\alpha)}{N(\text{Fe})}_{\text{Sun}}$

where $\alpha$ is an « $\alpha$-element », i.e. with a nucleus made of an integer number of $\alpha$-particles
How to determine $T_{\text{eff}}$

Multi-colour photometry

calibrated with stars having fundamentally determined $T_{\text{eff}}$

– Johnson’s UBV
– Strömgren’s uvby
– Geneva UBV B1 B2 V1 G

Spectroscopy

ratios of suitable strong lines
→ $T_{\text{eff}}$ or spectral type +
calibration $T_{\text{eff}}$ vs spectral type

excitation equilibrium

line profiles

$\vartheta_{\text{eff}} = \frac{5040}{T_{\text{eff}}}$

Source: Bessell 2005
IRFM: a semi-fundamental $T_{\text{eff}}$ scale

Empirical method, calibrated on the InfraRed Flux Method, gives a relation between photometric colours (like $V - I$, $V - K$) and $T_{\text{eff}}$.

Basic idea:

$$\frac{F(\text{surface})}{F_{\lambda\text{IR}}(\text{Earth})} = \frac{\sigma T_{\text{eff}}^4}{F_{\lambda\text{IR}}(\text{model})}$$

Once calibrated on stars with known diameters, any colour index can be calibrated on the IRFM.

Comparing different IRFM calibrations, the zero point proves to be uncertain by ±100K, in particular for metal-poor stars.

Source: Alonso et al. 1999
Photometry: $T_{\text{eff}}$ dependence

$T_{\text{eff}}$ variations dominate the flux variations of cool stars.

BB approximation: need to measure flux at 2 points to uniquely determine $T$.
In reality, $[m/Fe]$ and reddening complicate the derivation of photometric stellar params.
Spectroscopic $T_{\text{eff}}$ indicators: line-depth ratios (LDRs)

Ratio of two lines’ central depths can be very sensitive to temperature, if the lines are chosen to have different sensitivities to $T$

Ideally, the LDR is close to 1 and the lines should not be too far apart.

The main challenge lies in a proper $T_{\text{eff}}$ calibration across a usefully large part of the HR diagramme

Source: Gray, Fig. 14.7
Spectroscopic $T_{\text{eff}}$ indicators: excitation equilibrium

$T_{\text{eff}}$ is determined such that the abundance of an element (usually Fe) is independent of the excitation potential ($\chi_{\text{exc}}$) of the individual lines.

One needs many lines of a single element sampling a range of $\chi_{\text{exc}} \rightarrow$ iron.

Final precision depends on spectral resolution, choice and number of lines and S/N ratios.

\[ y = A + Bx = -0.87 \pm 0.14 + 0.006x \pm 0.037, \text{ rms}=0.217 \]

* Boltzmann’s eq. under LTE conditions

Source: Letarte, PhD
Spectroscopic $T_{\text{eff}}$ indicators: H lines

Wings of Balmer lines (above 5000K)

Cool stars: OK
Log $g$ and metallicity sensitivity is low, some dependence on the mixing-length parameter (H$\beta$ and higher).

Main challenge: recovering the intrinsic line profiles from (echelle) observations and proper normalization

Hot stars: Balmer lines can constrain the surface gravity

Source: Korn
How to determine log \( g \)

**Calibrated photometry**

\[
\log g_* = \log g_\odot + \log \frac{M_*}{M_\odot} + 4 \times \log \frac{T_{\text{eff}*}}{T_{\text{eff}\odot}} + 0.4 \times (M_{\text{Bol}*} - M_{\text{Bol}\odot})
\]

**Parallaxes** \( L, R, (M) \rightarrow g \)

**Spectroscopy**

Ratio of, e.g., Fe II to Fe I lines

Line profiles:
- B-to-F-type stars: Balmer
- Late-type stars: strong metallic lines with wings (e.g. Na ID doublet, Ca II IR triplet)

But there is always a temperature dependence - this must be secured beforehand or a pressure-temperature solution must be made.
Photometric log $g$

Largest gravity sensitivity: Balmer jump at 3647Å

Colour indices like $(U-B)$ or $(u-y)$ measure the Balmer discontinuity

Disadvantages:
- high line density in spectral region (missing opacity problem)
- difficulties with ground-based observations in the near-UV

The $c1$ index $[(u-b)-(b-y)]$ works well for metal-poor giants (Önehag et al. 2008)

Source: Gray, Fig.10.8
Spectroscopic log $g$: Balmer Lines

$H\gamma$: pressure indicator for $T_{\text{eff}} > 7500K$
Spectroscopic log \(g\): ionization balance

Measure of elements in two ionization states (e.g., Fe I & Fe II): both should yield same abundance, \(A\).

Gravity is a free parameter, to be varied until equality is reached.

By definition, gravity is related to \(P_g \propto g^{2/3}\) and \(P_e \propto g^{1/3}\).

In cool stars:
- Fe I, the dominant species, will depend on \(1/P_e\)
- Fe II, the minority species, with majority of atoms in state \(i - 1 = 1\), will depend on \(1/P_e\)

\[\Delta \log \varepsilon = 0.1 \text{dex} \rightarrow \Delta \log g = 0.3 \text{dex} \quad (0.1 \text{dex} \approx \text{line-to-line scatter})\]
Spectroscopic log $g$: the strong line method

Damped (neutral) lines show a strong gravity sensitivity, because

$$L_v \propto \gamma \propto P_e / g^{2/3}$$

Like with ionization equilibria, log $\varepsilon$ needs to be known

(best if obtained from weak lines of the same ionization stage, preferably originating from the same lower state:

Ca I 6162, Fe I 4383, Mg I 5183, Ca I 4226)

Below $[\text{Fe/H}] \approx -2$, there are no optical lines strong enough to serve as a surface-gravity indicator
How to determine the microturbulence velocity, $\xi$

Observed EW of saturated lines > predicted values using thermal and natural broadening alone $\rightarrow$ extra broadening introduced, the micro-turbulent velocity $\xi$ (fudge factor)

Caused by small cells of motion in the photosphere (treated like an additional thermal velocity in the line absorption coefficient)

No effect on weak lines: these are gaussian, broadening them also makes them shallower $\rightarrow$ EW is preserved

Can be important in strong lines, by broadening and hence de-saturating them.

It is usually determined by ensuring that for individual elements EW is independent of line. Typical values are 1-2 km/s.

Source: Letarte, PhD
How to determine the metallicity

Calibrated photometry

After $T_{\text{eff}}$ (and maybe reddening) the global metallicity has largest influence on stellar flux

Difficult for stars with $[\text{Fe/H}] < -2$ (e.g. $\delta(U-B)$ loose sensitivity)

Limited precision: $\approx 0.3$ dex

Spectroscopy

Equivalent widths
## Pros vs. Cons

### Photometry

- ✓ an efficient way of determining stellar parameters
- ✓ can probe very deep
- ✓ freely available (surveys!)
- ✓ comparatively cheap to obtain.

- ✗ limited (which parameters can be derived)
- ✗ subject to extra parameters (reddening!)
- ✗ subject to parameters that cannot be determined well ($\xi$, $[\alpha/Fe]$).

### Spectroscopy

- ✓ a way of determining a great number of stellar parameters
- ✓ the key technique for obtaining detailed chemical abundances
- ✓ (usually) reddening-free.

- ✗ comparatively costly at the telescope
- ✗ currently limited in $V$
- ✗ more complex to master.
• Stellar atmospheres
• Characteristics of star → stellar parameters
• **From lines to abundances**
  • Lines (atomic and/or molecular)
  • Model atmospheres
  • Available tools
From spectral lines to abundances

Curve of growth

Spectrum synthesis

Differential analysis:
→ Comparison of one star to another
→ Ratio of abundances
→ Reference star is necessary
From spectral lines to abundances

Different methods and tools, but ...

Need to relate intensity/strength of an observed line to amount of corresponding element in original stellar gas, i.e. need to find the number of absorbing atoms per unit area ($N_a$) that have electrons in the proper orbital to absorb a photon at the wavelength of the spectral line

Boltzmann and Saha equations are applied and combined with the pressure and temperature of the gas to derive an abundance of the element (i.e. to calculate excitation and ionization):

Saha’s equation

$$\frac{n_1}{n_0} P_e = \frac{(2\pi m_e)^{3/2} (kT)^{5/2}}{h^3} \frac{2u_1(T)}{u_0(T)} e^{-\frac{1}{kT}}$$

It gives the relative number of atoms in two ionization states as a function of electron density $n_e$ and temperature $T$

Boltzmann’s equation

$$\frac{N_b}{N_a} = \left( \frac{g_b}{g_a} \right) \left( e^{-\frac{(E_b - E_a)}{kT}} \right)$$

It gives the population of energy levels $a$ and $b$
From spectral lines to abundances

However, not all transitions between atomic states are equally likely. Each transition has a relative probability, or $f$-value (also called oscillator strength).

Can be calculated theoretically or measured in a lab

Number of atoms lying above each cm$^2$ of photosphere: $N_a \times f$-value

\[ \frac{F_\lambda}{F_c} \]

\[ 0 \quad 0.3 \quad -0.1 \quad 0 \quad 0.1 \quad 0.2 \quad 0.3 \]

$\Delta \lambda$ (Å)

**Figure 9.20** Voigt profiles of the K line of Ca II. The shallowest line is produced by $N_a = 3.4 \times 10^{11}$ ions cm$^{-2}$, and the ions are ten times more abundant for each successively broader line. (Adapted from Novotny, *Introduction to Stellar Atmospheres and Interiors*, Oxford University Press, New York, 1973.)
Measuring abundances: equivalent width

Equivalent width:

\[ W_\lambda = \int_0^\infty \frac{I_\lambda - I_c}{I_c} \, d\lambda \]

Geometrically:

\[ W_\lambda = \text{width of a rectangular, completely opaque line} \]

For a 2-component atmosphere (continuum source + cool layer of depth h), the rectified line profile is:

\[ r(\lambda) = \frac{I_\lambda(0) e^{-[\kappa_\lambda(0) + \kappa_c(\lambda)]h}}{I_c(0) e^{-\kappa_c(\lambda) h}} = e^{-\kappa_\lambda(\lambda) h} \]

Then

\[ W_\lambda = \int_0^\infty [1 - r(\lambda)] \, d\lambda \]

For faint lines:

\[ W_\lambda \approx \int_0^\infty \kappa_\lambda(\lambda) \, h \, d\lambda = Nh \int_0^\infty \alpha_\lambda \, d\lambda = Nh f \frac{\pi e^2}{mc} \frac{\lambda^2}{c^3} \ll Nh f \lambda^2 \]

where N is the number density of atoms able to absorb the incident radiation
Measuring abundances: curve of growth

Tool to determine $N_a \rightarrow$ abundance (EW varies with $N_a$)

**Weak lines:** linear part \[ W \propto A \]

Doppler core dominates and the width is set by the thermal broadening $\Delta\lambda_D$. Depth of the line grows proportionally to abundance $A$

**Saturation:** plateau \[ W \propto \sqrt{\log A} \]

Doppler core approaches max. value and line saturates towards a constant value

**Strong lines:** damping \[ W \propto \sqrt{A} \]

wings dominate

optical depth in wings becomes significant compared to $\kappa_\nu$. Strength depends on $g$, but for constant $g$ the EW is proportional to $A^{1/2}$

**Small/weak lines are best for abundance determination**
**CoG dependencies: temperature**

Temperature can affect line strength via:

- \( N_r/N_E, \kappa, \theta_{ex} \)

CoG shape looks the same, only shifted for different values of the excitation potential.

Fewer atoms are excited to the absorbing level when \( \chi \) higher.

Amount of each shift can be interpreted as \( \theta_{exc} \chi \).

**Fig. 16.1.** Curves of growth shift to the right as the excitation potential is increased. These curves are computed for Fe I \( \lambda 6253 \) in a solar-temperature model with a surface gravity of \( 10^4 \text{cm/s}^2 \). When the shifts are translated into \( \theta_{ex} \) according to Eq. (16.4), \( \theta_{ex} \) is found to vary slightly as shown in the inset graph; higher excitation lines are formed deeper where the temperatures are higher. The dashed curves show scaling for a fixed \( \theta_{ex} \) of 1.01, and we see that the differences in the linear portion are considerably smaller than observational errors. Differences in development of the strong-line portion of the curves stems from the increase in van der Waals damping with increasing excitation potential.
CoG dependencies: gravity

Gravity can affect line strength through:

- $N_r/N_E$ (if increases, $A$ decreases at given $W/\lambda$)
- $\kappa_v$

Since both of these can be sensitive to the pressure, for neutral lines the effects cancel $\rightarrow$ effect important only for ionized species
CoG dependencies: microturbulence

The presence of microturbulence delays saturation by spreading the line absorption over a larger spectral band.
CoG analysis for abundances

Advantages:

Simple, you measure the equivalent width of a line and read the abundance off the log $W$ vs log $A$ plot

Disadvantages:

Lots of calculations
Difficulty in dealing with microturbulence and saturation effects:

Make an initial guess of $\xi$
Theoretical cogs are calculated for all measured EWs of some element with lots of lines
From each line one derives an abundance $A$ and plots it vs $W$
If $A$ is found to be a function of $W \rightarrow \xi$ must be wrong

One happily choose a new $\xi$ and start all over

This must continue until one finds convergence
CoG: principles of abundance estimate

1. Compute a theoretical cog for a given line of a given atom/ion

Observe $W$ of this line $\rightarrow \log A_0$

This is what codes do implicitly for each line

2. If one doesn’t want to compute more than 1 cog:

Observed $W$ of other lines

Then plot $\log(W/\lambda)$ vs $\Delta \log A$

Then empirical cog

Then shift curve onto theoretical one

$\Delta \log A = \log \left( \frac{g_f \lambda}{g_0 f_0 \lambda_0} \right) - \theta(m (X - X_0))$

$\rightarrow$ shift=$\log A_0$
Crowding & Blends

→ Spectrum Synthesis
In real life, one no longer does a curve-of-growth analysis, but rather a full spectral synthesis.

Fig. 16.9. The circles show the observed spectrum, while the lines are for models ($T_{\text{eff}} = 4725$ K, log $g = 1.70$, and $\xi = 1.60$ km/s) with different chemical abundances. The solid line is deemed to fit best. Based on data in Fig. 2 of Burris et al. (2000). The resolving power is $\sim 20,000$ and the signal-to-noise ratio $\sim 100$. 

Spectrum synthesis

Source: Heiter
Measuring abundances: expected precision

It is difficult to determine the temperature of a star to better than 50–100 K

It is difficult to determine the gravity of a star to better than 0.1-0.2 dex

It is difficult to determine the microturbulent velocity of a star to better than 0.2 km/s

**Absolute** abundances $> 0.1$-0.2 dex!

**Problems**
- Uncertain or wrong log(gf) values
- NLTE effects
- 3D hydrodynamic models → $Z_{\text{Sun}} = 0.012$ instead of 0.018!

**But**

One can work differentially, by taking the abundance ratios between two similar stars ($\approx T_{\text{eff}}$) → uncertainty on the oscillator strengths cancel.

**Differential** abundances (rel. to the Sun) $\approx 0.04$-0.05 dex!!
• Stellar atmospheres
• Characteristics of star → stellar parameters
• From lines to abundances
• **Lines (atomic and/or molecular)**
• Model atmospheres
• Available tools
Line List

A proper line list is a critical part of the analysis, and building one needs some care. Lines need to be chosen carefully, making sure they have reliable gf-values and are sufficiently isolated from their neighbours at the resolution of the observations and of course to lie within the wavelength coverage of the instrument.
Line absorption data

Need to know: $\lambda$, $E_{\text{low}}$, $J_{\text{low}}$, $f$, $\gamma_1$, $\gamma_2$

NIST Atomic Spectra Database
http://physics.nist.gov/PhysRefData/ASD/lines_form.html

VALD Vienna Atomic Line Database

HITRAN HIgh-resolution TRANsmission molecular absorption database
http://www.cfa.harvard.edu/HITRAN/

Not to forget all the missing (or not well defined) lines ....
• Stellar atmospheres
• Characteristics of star → stellar parameters
• From lines to abundances
• Lines (atomic and/or molecular)
• Model atmospheres
• Available tools
What is a model stellar photosphere?

To make the modelling of stellar atmospheres manageable, a variety of assumptions are traditionally made about stellar photospheres.

Model atmosphere

- Plane parallel layers along depth \( t \)
- Parameters:
  - Gravity \( g \) (constant)
  - Effective temperature \( T_{\text{eff}} \)
  - Chemical composition \([\text{M/H}]\)
- Equations:
  - Transport of radiative energy
    \[
    \frac{dI}{ds} = \varepsilon - \kappa \cdot I
    \]
  - Conservation of energy
    \[
    F(t) = \sigma \cdot T_{\text{eff}}^4
    \]
  - Hydrostatic equilibrium
    \[
    \frac{dP}{dt} = g \cdot \rho
    \]

Source: Heiter
Assumptions and consequences

(1) plane-parallel geometry
   all physical variables a function of only one space coordinate
   but stars are essentially spherical!
   in many cases photosphere is very thin: $\Delta R / R \ll 1 \,(\approx 5 \times 10^{-4} \text{ Sun})$

(2) homogeneity
   no fine structures and granularity
   but stars with convection, spots, etc ... difficult to resolve
   average homogeneous model $\Rightarrow$ average stellar properties

(3) Stationarity
   time-independence of stellar spectra on human timescales
   plenty of exceptions here (pulsations, mass loss, SN)
   hope to observe average stellar properties

Source: Radiative Transfer in Stellar Atmospheres, R.J. Rutten 2003
What is a model stellar photosphere?

(4) hydrostatic equilibrium
no large scale accelerations in photosphere
no dynamical significant mass loss

gravitational force: \[ dF_{\text{grav}} = -G \frac{M \, dm}{r^2} = -g(r) \, dm \]
pressure force: \[ dF_p = -A(P(r+dr) - P(r)) \]
radiation force: \[ dF_{\text{rad}} = g_{\text{rad}} dm \] (mostly important in hot stars \( \alpha T^4 \))

\[ \Sigma dF_i = 0 \Rightarrow \frac{dP}{dr} = -\rho(r) \left( g(r) - g_{\text{rad}} \right) \]

(5) flux constancy (radiative equilibrium)
stellar atmospheres are much too cool and tenuous to fuse nuclei
energy coming from core just transported by radiation or convection
total energy output (luminosity):
\[ L = 4\pi r^2 F(r) = \text{const.} \]

(6) local thermodynamic equilibrium
existence of a radial temperature gradient \( \Rightarrow \) 2 volumes not in eq.
validity of TE locally

Source: Radiative Transfer in Stellar Atmospheres, R.J. Rutten 2003
Model outputs

A 1D model atmosphere is a tabulation of various quantities as a function of (optical) depth.

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<th>$\log \tau_0$</th>
<th>$T$ (K)</th>
<th>$\log P_g$ (dyne/cm$^2$)</th>
<th>$\log P_e$ (dyne/cm$^2$)</th>
<th>$\log \kappa_0/P_e$ (cm$^2$/g per dyne/cm$^2$)</th>
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Table 9.2. Model photospheres.
• Stellar atmospheres
• Characteristics of star → stellar parameters
• From lines to abundances
• Lines (atomic and/or molecular)
• Model atmospheres

• Available tools
Data and tools needed

Data needed:

– **Model atmospheres** (esp. T-τ relation) for various $T_{\text{eff}}$, logg and chemical compositions

  • Kurucz models (ETL, plane parallel): very extended grid, 
    http://kurucz.harvard.edu/grids.html

  • (OS)MARCS models (ETL, plane parallel/spherical): for cool stars (4000 to 8000 K) 
    http://marcs.astro.uu.se/

  • TLUSTY models (NLTE, plane parallel): for hot stars (27500 to 55000 K) 
    http://nova.astroumd.edu/Tlusty2002/tlusty-grids.html

– **Line data**: wavelengths, excitation potentials, oscillator strengths, broadening parameters


– **Observed spectrum** normalized to the continuum, and equivalent widths
Tools needed:

- **Data reduction software**
  - Iraf, MIDAS and/or instrument-specific pipelines, IDL

- **Good procedure for continuum normalization** → not so easy (e.g. Hα)

- **Code of spectral synthesis**, e.g.:
  - ATLAS/SYNTHE (Kurucz)
    - [http://kurucz.harvard.edu](http://kurucz.harvard.edu)
  - Moog (Sneden) for average and cool stars
    - [http://verdi.as.utexas.edu/moog.html](http://verdi.as.utexas.edu/moog.html)
  - (OS)MARCS (Uppsala/B. Plez suites)
    - [http://marcs.astro.uu.se/](http://marcs.astro.uu.se/)
  - SME (Valenti & Piskunov)
    - [http://tauceti.sfsu.edu/Tutorials.html](http://tauceti.sfsu.edu/Tutorials.html)
  - Synspec (Hubeny & Lanz) for hot stars
MARCS Stellar Models

Plez 2000-2002

- Geometry: Plane-parallel approximation
- Temperature: $3800 \leq T_{\text{eff}} \leq 5200$ K in steps of 200 K
- Gravity: $0.5 \leq \log g \leq 4.5$ dex in steps of 0.5 dex
- Metallicity: $-4.0 \leq [\text{Fe/H}] \leq -1.00$ dex in steps of 0.25 dex
- Alpha: Enhanced, $[\alpha/\text{Fe}] = 0.4$

MARCS 2005

- Geometry: Spherical
- Temperature: $4000 \leq T_{\text{eff}} \leq 5500$ K in steps of 250 K
- Gravity: $0.0 \leq \log g \leq 3.5$ dex in steps of 0.5 dex
- Metallicity: $-1.5 \leq [\text{Fe/H}] \leq +1.00$ dex in steps of 0.25 dex
- Alpha: Standard, $[\alpha/\text{Fe}] = 0$ at $[\text{Fe/H}] = 0$, $+0.1$ for each -0.25 dex until it reaches $+0.4$ at $[\text{Fe/H}] \leq -1.0$.

Plez 2005

- Geometry: Spherical
- Temperature: $3600 \leq T_{\text{eff}} \leq 4000$ K in steps of 200 K
- Gravity: same as MARCS 2005
- Metallicity: $-3.0 \leq [\text{Fe/H}] \leq -1.50$ dex in steps of 0.5 dex
- Alpha: Poor, $[\alpha/\text{Fe}] = 0.00$ for all models.
The Solar Spectrum

Source: Kurucz