A

δ Scuti studies with the STARE data

We study a particular case of a δ Sct star (V1821 Cygni) observed by the STARE telescope in a field centered in the constellation of Cygnus. This star had been observed by several authors to be multi-periodic, and there were clues for variations in the amplitudes of one of its modes. The light curve obtained with STARE is used to confirm this amplitude variability, to investigate on its origin (close frequencies vs. intrinsical mode variability), and to detect previously unseen low amplitude pulsation modes. This provides an example of the use of the STARE (and TrES) light curve databases to perform investigations on stellar pulsations that can compete with those that use much bigger aperture telescopes.

A.1 Introduction

V1821 Cygni is a δScuti type pulsator that was discovered in a search for variable stars in the open cluster NGC 6871 (Delgado et al. 1984, D84 hereafter). It was found by these authors that the star is not a member of the cluster, but a foreground object. A period analysis of their set of 3 observing nights allowed them to identify two frequencies, $f_1 = 9.245$ and $f_2 = 11.076$ c/d. A second intensive study of this star was done by Zhou, Liu, & Du (2001), observing a total of 21 nights spanning 354 days. In their period analysis, they detected two different frequencies, 8.8218 and 8.2439 c/d. These authors also re-analyzed the D84 set of data, obtaining two terms at 8.9788 and 11.0674 c/d. The different results in the period analysis of the same set of data is a consequence of the short duration of the series, according to the authors. They

noticed that a 1 day alias of the D84 f_1 roughly coincides with the 8.2439 c/d frequency. A fit of the D84 data with the frequencies obtained by Zhou, Liu, & Du (2001), allowing for phase and amplitude variations, was carried by these authors, and they observed an amplitude variability on the 8.8218 c/d frequency. No evidence was found for a frequency close to 11.076 c/d in the new set of data, making them suggest that, if this frequency is real, it has to be produced by an unstable mode.

The issue of amplitude and phase variations in several observed modes (in XX Pyx, Handler et al. 1998, Handler et al. 2000; V1162 Ori, Arentoft et al. 2001) is a controversial point. It is not well established if these observed values are produced by the effect of close frequency pairs (as suggested by Alvarez et al. 1998 for the case of BW Cnc), or if the variations are intrinsic to each mode. To illustrate the difficulties in disentangling these two interpretations, we mention a study carried by Handler et al. on XX Pyx in 1998. This star seemed to show long-term variations in the amplitude of the highest amplitude frequency f_1, that had a different time scale than the phase variations of the same mode. However, a new set of observations (Handler et al. 2000), allowed them to identify a new frequency close to f_1, that was the final cause of their detected frequency and amplitude variability. After developing and applying a non-linear analysis to the series, they claimed to find evidences for short-term amplitude and frequency variations in other 8 modes of the star.

Breger & Bischof (2002) performed a literature search for detected close frequencies in δ Scuti stars, and for one of these stars (BI CMi), they found phase variations in three of the modes when their amplitude reaches a minimum, thus favoring the close-frequency model instead of simple amplitude variability. They provide several possible explanations for these close-frequency pairs: mixed modes, trapped modes, rotational splitting, small spacing, and mode coupling. Some of these explanations (small spacing, trapped modes) can account for two detected close frequencies, but can not explain a group of three or more close frequencies.

A.2 Observations

The observations were obtained as a by-product of a planetary transit search in a field centered in HD 188650, carried by the STARE telescope (Chapter 2). A field containing V1821 Cygni was observed during summer 2001, for a total duration of 205 hours, obtained in 36 nights spanning 87 days. The filter used was roughly an R filter, and the exposure time was fixed at 107 s, producing typically one point every two minutes, due to the readout time. As a final result of the data reduction, there are a total of 820 points, sampled every

TABLE A.1— The light curves. N stands for number of data points. JD0 is Julian Date 2450000.

ID	N	Begins (JD - JD0)	Ends (JD - JD0)	Spans (days)
1	112	2136.5846	2143.7245	7.14
2	162	2145.4089	2151.4952	6.09
3	154	2153.4424	2162.5291	9.09
4	167	2165.3411	2172.5640	7.22
5	167	2183.3684	2205.5394	22.17
6	161	2487.3917	2493.7206	6.33

15 min. A good night of observation, sampled every two minutes, is plotted in Figure A.1. The amplitude spectrum of one close, 0.5 mag fainter constant star (HD 227732) is plotted in Figure A.2, to evaluate the precision level attained. It can be seen that there are no peaks detected above the 1 mmag level.

To study possible variations in amplitude of some of the frequencies (as stated in Zhou, Liu, & Du 2001), we made a subdivision of the data set, in an arbitrary number of 5 groups. This was done subjectively, trying to keep good runs of $\sim$ 6 days together, and, when not possible, trying to obtain enough data points. The length, number of points, and time span of each subdivision is summarized in Table A.1. The total number of data points is 762. The discrepancy between this value and that stated above (820) is due to three nights of data at the beginning of the observing run. These nights were too isolated in time (15 days) from the rest of the time series, and thus not considered for the study of the variations in the frequencies and amplitudes. The subdivision *5* in that table spans longer than the others. We made this subdivision trying to get enough points to allow a spectral study, thus compensating for the short durations of each nightly observation.

V1821 Cygni is the only cataloged δ Scuti star in this field, but several non-cataloged δ Scuti stars were discovered in the same field (Alonso, Belmonte & Brown 2003, see Figure A.3). Some of these stars will be used in this study to check for amplitude and frequency variations, allowing us to detect possible systematic errors in the data acquisition and/or analysis.

A.2.1 The 2002 data set

During summer 2002, the same field was re-observed. Unfortunately, due to technical problems, only 16 nights in a total span of 78 days were useful. Of these, the best set of data consists of 6 nights spanning 7 days, that will be the time series used for our purposes. The rest of the nights are too isolated in time; we discarded them to avoid complicated aliasing in the spectra.

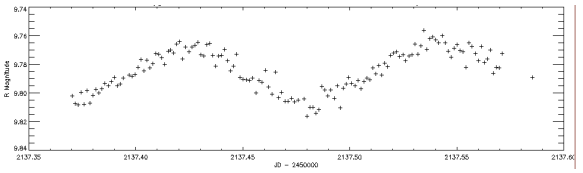

FIGURE A.1— Data from a night of observation on V1821 Cygni

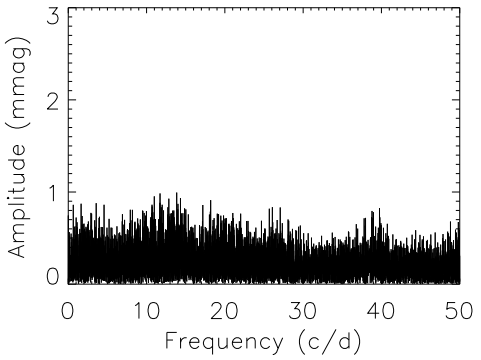

FIGURE A.2— Spectral analysis of a close, 0.5 mag fainter constant star (HD 227732, A0 type)

A.3 Data analysis

The frequency analysis was carried out with the tool Period98 (Sperl 1998), for consistency with the work of Zhou, Liu, & Du (2001). As a first analysis, we used the combined 2001-2002 time series to establish the best values of the frequencies, amplitudes and phases that matched with the observations. Table A.2 summarizes the results of this analysis. Zhou, Liu, & Du (2001) mention a possible amplitude variability of the frequency f_1, that is specially noticeable in the different time series of the star obtained by Delgado et al. (1984) and their new set of data. At this point, we should mention that both set of observations could be compared because they were obtained with the same Strömgren y filter. Our data cannot be directly compared to these results, as we used a roughly Johnson's R filter. The frequency resolution in the 2001 set of data is 0.011 c/d, if $1/\Delta T$ is used (ΔT is the span of the observations), or 0.016 c/d if the more conservative Loumos & Deeming (1978) approach is used.

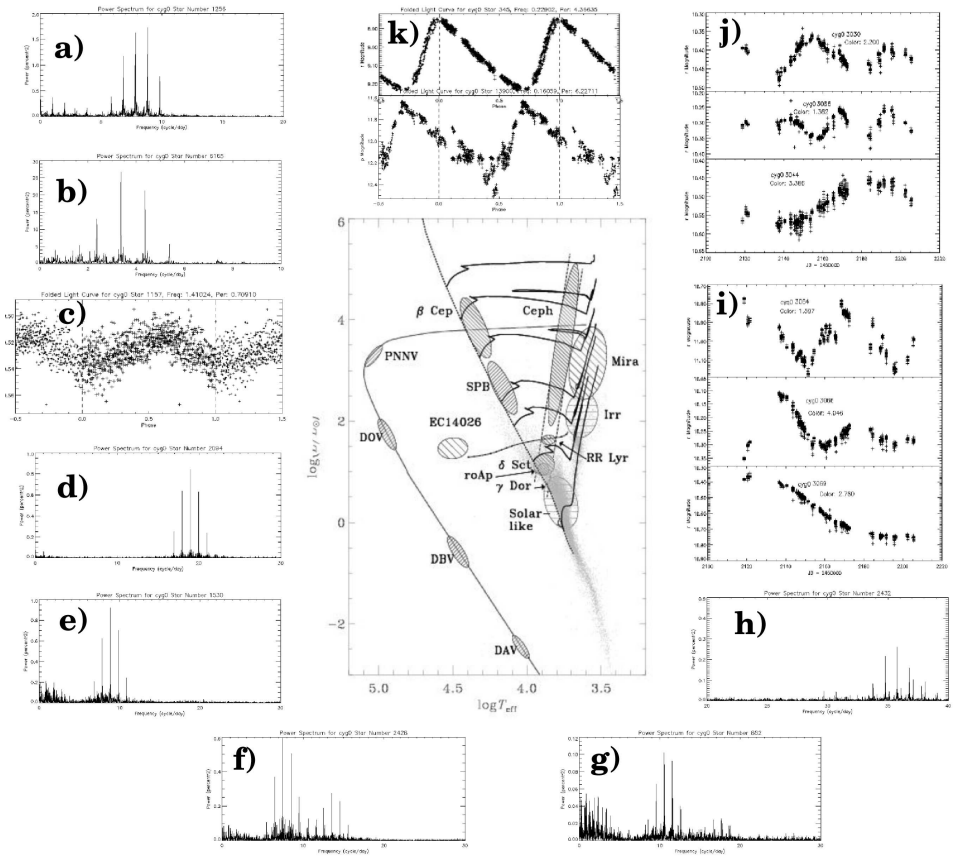

FIGURE A.3— Different examples of variability across the H-R diagram, from STARE data. a) Power spectrum of HD227977, probably a βCepheid star. b) and c) Two stars with oscillations in the range [1-5] cycles/day, thus probably γDoradus or SPBs. d),e),f),g),h) Power spectra of stars with oscillation periods in the range [6-40] cycles/day; d) shows a monoperiodic behavior, f) and g) show a complex spectrum, and h) shows frequencies higher than 30 cycles/day. i) and j) are plots of the light curves of long period or irregular red giant stars. k) are two folded light curves of classical Cepheids. Top: V547Cyg. Bottom: V402Cyg. The figure at the center is from Christensen-Dalsgaard (1999). From Alonso, Belmonte & Brown (2003).

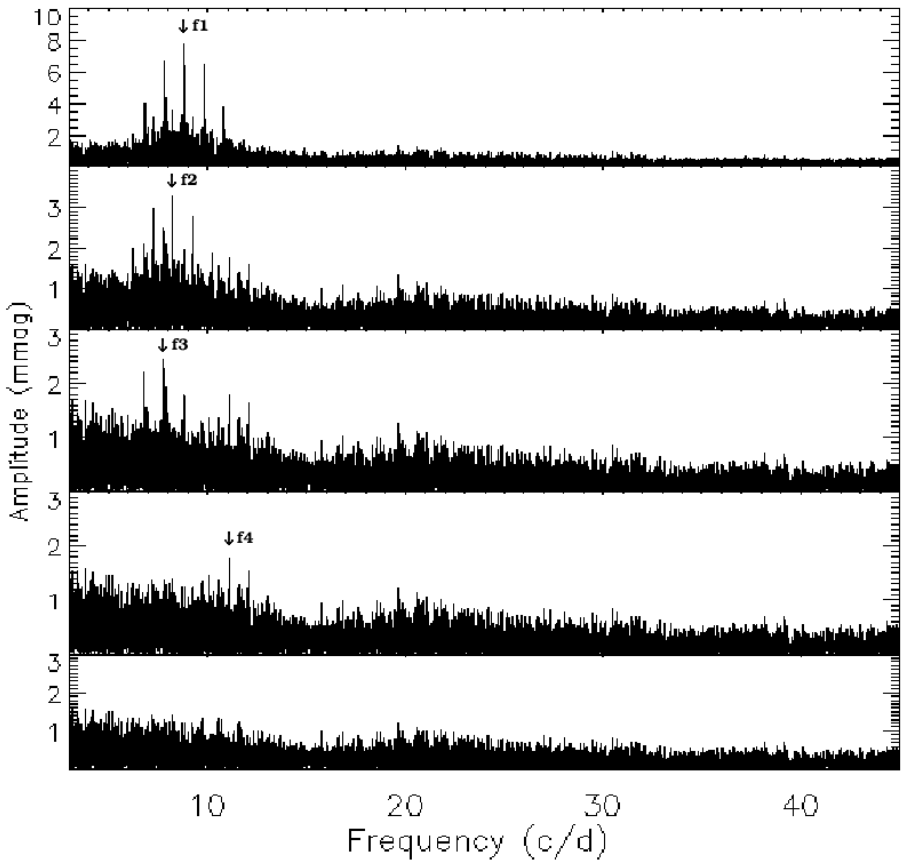

FIGURE A.4— Amplitude spectra of the 2001-2002 time series of V1821 in the region 3-45 c/d, obtained with Period98, after successive pre-whitening of the frequencies presented in Table A.2.

TABLE A.2— Results of the pre-whitening analysis. The noise was calculated in the residual spectra, as the mean amplitude level inside a 5 c/d box centered at each frequency. The frequency resolution for the 2001 data set is ~ 0.016 c/d (see text for details).

ID	Frequency (c/d)	Amplitude (mmag)	S/N
f_1	8.819	8.4	17.8
f_2	8.243	3.4	6.9
f_3	7.775	2.6	5.3
f_4	11.086	1.8	4.4

A.3.1 Amplitude variability

To see if the amplitude variability reported by these authors is also present in our data, we used the different sets of data described in Table A.1. In each subset, we forced a least-squares fit to the data with the frequencies f_1, f_2, f_3 and f_4 fixed, letting the amplitudes and phases of these frequencies vary. The result of the standard deviations from the mean of the amplitude and phase of f_1 in each subset is respectively 1.69 mmag and 0.037 cycles. We need to determine if this variation is at the level of our detection capabilities. To accomplish this, in the next paragraphs we will remove the influence of f_1 in the light curve, and introduce a signal with the value of f_1, but with constant amplitude and phase. The results of the least-squares fit, as performed above, will give us an idea of the dispersions in amplitude and phase that we are able to detect.

To remove the signal of f_1, we made a least-squares fit to each subset. The fixed parameters were the frequencies f_2,f_3 and f_4, and the fitted parameters were the amplitudes and phases of these frequencies, together with the amplitude, phase and frequency f_1. We subtracted from the time series a sine wave with the values of the fitted frequency f_{1b}, amplitude and phase. Putting together all the subseries results in a time series with the influence of f_1 and possible variations in its amplitude and phase (i.e. frequency) removed.

As the next step, we added an artificial sine wave with fixed frequency, amplitude and phase to the whole (2001-2002) time series. We repeated the fit done to the initial light curve to this new time series, subdivided in the same subsets. The result is a much lower dispersion in the fitted values for the amplitudes and phases than the obtained with the real data: 0.19 mmag for the amplitude and 0.007 cycles for the phase.

It could be argued that with an a-priori knowledge of the frequency f_1 to be adjusted, it is obviously expected that the fit of this artificial light curve is better than the fit to the real data. To see if our results are affected by this effect, we made two different fits to the subseries, with a fixed frequency lower than the artificial inserted frequency ($f_1' = 8.815$ c/d instead of 8.81853 c/d) and with a frequency higher ($f_1'' = 8.82179$ c/d, the value reported in Zhou, Liu, & Du (2001). The standard deviations of the amplitudes and phases of the fitted f_1' and f_1'' are 0.33 and 0.18 mmag, and 0.068 and 0.092 cycles, respectively. The result of these fits shows that while an incorrect estimation of the frequency f_1 causes a linear drift or decrease in the fitted phase (and consequently a bigger dispersion in the measured phases), it does not produce variations in the amplitude of the order of the observed ones.

A.3.2 Phase variability

Once established that there are amplitude variations in f_1, we initiated a search for possible phase variability in this frequency. To do this, we need to model the amplitude variations of the frequency f_1. As a first approach, we will assume that the amplitude variability can be well described by a sinusoidal modulation. We used the measured amplitude and phase of f_1 in each subset, as described in the previous subsection, to calculate the parameters of this modulating sine wave. The analysis with p98 returned a peak at a frequency of 0.0284 c/d (35.1854 d), amplitude of 1.83 mag, and phase of 0.49 cycles. This new frequency was used to modulate the amplitude of an artificial sine wave with frequency f_1. We added this signal to the time series, described above, with the influence of the *real* f_1 removed.

Finally, we performed least-squares fits to this new time series, in each of the subsets described in Table A.1. In these fits, as before, the fitted parameters were the amplitude and phases of the fixed 4 frequencies. In this case, the standard deviations of the amplitude and phases resulted in 1.28 mmag and 0.005 cycles, respectively. This shows that, while a sinusoidal variation of the amplitude of f_1 can account for the dispersion in the amplitude, the dispersion in the phase is not so big as in the real data. Once again, a similar analysis carried out with a fit to the frequency f_1 with an incorrect value causes the phase to change monotonically, which is not the case in the real data. This result motivated a more detailed analysis on the amplitude and phase behavior of V1821, that will be described in Subsection A.3.4.

A.3.3 Is the phase variation instrumental?

There is still another point to be checked before claiming a phase (or frequency) variation: could there be some instrumental problem in the data acquirement or data reduction pipeline that produces false phase variations? For instance, errors in the data acquisition PC clock (which is GPS-updated every hour), or truncations in the Julian Date vectors might be a source of phase variation. To test this possibility, we used other variable stars that are in the same field, with similar magnitude, and variations with similar frequencies as V1821 Cygni. These stars will have data at the same times as V1821 Cygni, and, if there were some instrumental problem with the times, all of them should reflect a similar behavior. As these stars were discovered in this run, there is no available information on their variability. Table A.3 lists these stars and the results of the frequency analysis on the 2001 data set.

For each star, we performed the same analysis as for V1821:

- Find the frequencies in the 2001 data set.

TABLE A.3— New δ Scuti stars in the field of V1821, used to test for amplitude and phase variability. When there is no ID of the star, the 2MASS coordinates are provided.

STARE ID	ID	RA (2000.0)	Dec (2000.0)	Frequency (c/d)	Amplitude (mmag)	Phase (cycles)
8267	-	$19^h55^m23.80^s$	37°28'59.4''	7.7073	65	0.50
				15.4142	20	0.31
2094	HD 225708	$19^h46^m07^s$	35°25'07''	18.9369	8.8	0.84
				27.2694	1.6	0.77
				21.1748	1.6	0.99
8663	-	$19^h48^m11.03^s$	37°01'21.2''	5.3137	7.1	0.95
				3.9744	2.3	0.74
				5.0191	2.1	0.97
1819	BD+36 3770	$19^h55^m17^s$	37°21'30''	15.8649	4.9	0.39
				18.2048	1.6	0.79
4566	-	$19^h40^m56.30^s$	36°04'10.3''	8.8316	13.2	0.82
				9.0416	2.1	0.20

- Construct the subseries.
- Fit the amplitudes and phases in each subseries, fixing the frequencies.
- Calculate the standard deviations of the amplitudes and phases in all the subseries.

The result of this analysis is summarized in Table A.4. Even if each star shows a different behavior, possibly due to the contribution of lower amplitude modes not taken into account, it can be seen that the standard deviation from the mean of the phase is around a value of $\sim$ 0.018 cycles, except for two cases: STARE 8663 and V1821 Cygni. The values of the standard deviations in the amplitude and phase of 8663 seem to indicate that this star also shows amplitude and phase variations, as we will demonstrate in the next subsection. The higher dispersion of the phase measurements in these two cases is a good proof of some phase (i.e. frequency) changes taking place in these stars, and disproves a possible instrumental origin of this result.

In the next section, we will try to disentangle if these amplitude and phase variations are intrinsic to the peak at f_1, or if they are caused by two or more close frequencies not resolved in each subseries.

A.3.4 A detailed study of the amplitude and phase variations

There are several possible effects that can produce amplitudes and phases variations. Among these, two close frequencies can in several cases explain these

TABLE A.4— Means and standard deviations of the amplitudes and phases for the fitted frequencies of the stars in Table A.3 to each subserie. See text for details.

STARE ID	Mean Amplitude	std amp	Mean phas	std phase
8267	67	0.73	0.50	0.017
2094	8.8	0.79	0.83	0.019
8663	7.2	1.56	0.95	0.031
1819	5.0	1.00	0.39	0.003
4566	13.1	1.15	0.82	0.012
V1821	7.8	1.57	0.82	0.026

variations (Breger & Bischof 2002), and favor this scheme against the single mode with variable amplitude hypothesis. A known effect of two close frequencies is a phase change that is steeper when the beating of the two frequencies reaches a minimum (see, for instance, Breger 1981). The amount of the phase change depends on the relative amplitudes of the two close modes. To try to establish which of these two favored explanations (single mode with variable amplitude vs. close modes) matches our data better, we made an amplitude - phase analysis, strongly inspired by the works of Arentoft et al. (2001) and Handler et al. (1998).

To avoid effects arising from the low amplitude frequencies, the time series first has to be cleaned of these. The way we accomplished this is as follows: first, in each subset described in Table A.1 we made a fit of the data to f_1,...,f_4. In this fit, the free parameters were the amplitudes and phases of f_2,f_3 and f_4, and the frequency, amplitude and phase of f_1. This ensures that we are removing the most powerful peak in the power spectrum of the subset. A synthetic light curve with the frequency, amplitude and phase of the best fit to f_1 in each subset was created and subtracted from the subset. Secondly, we put together these subsets having removed the contribution of f_1, and performed a frequency analysis, which gave a clearer fit to the low amplitude frequencies. The frequencies, amplitudes and phases of the four highest amplitude peaks coming out from this fit are in Table A.5. As a result of this analysis, a new frequency $f_{spurious}$ was detected, at a S/N level of 3.3. The value of this frequency, close to 7 c/d (at a distance lower than twice the spectral resolution), its lower S/N ratio, and the fact that it was not detected in the analysis carried out in Section A.2, makes us suspect that this is an spurious peak. The rest of the frequencies agree reasonably well with those tabulated in Table A.2, considering the spectral resolution (0.011 c/d). The final step was to construct a synthetic light curve with these low amplitude frequencies and subtract it from the original data. A frequency analysis of this last light curve shows a single peak at f_1, with the

TABLE A.5— Results of the pre-whitening analysis of the time series with the influence of f_1 removed. The noise was calculated in the residual spectra, as the mean amplitude level inside a 5 c/d box centered at each frequency.

ID	Frequency (c/d)	Amplitude (mmag)	S/N
f_2	8.241	3.3	7.1
f_3	7.773	2.5	5.3
f_4	10.082	1.5	3.8
$f_{spurious}$	6.980	1.6	3.3

low amplitude frequencies well removed.

To study the amplitude and phase variations with enough temporal resolution in this time series, we performed a *moving fit* to different subseries of the data. To construct these subseries, we fixed a duration (in time) and a step (in number of points) between two consecutive subseries. This ensures that all the subseries will have similar spectral resolution. We ended with $\sim N/step$ dependent subseries. In each of these subseries, we performed a 2 free-parameters (amplitude and phase) non-linear fit to a sine with frequency 8.818716277 c/d using the Levenberg-Marquardt algorithm (Levenberg 1944 and Marquardt 1963). We assigned the results of this fit (amplitude and phase) to the mean value of the time in each subseries. In Figure A.5 we plot the result, where the duration of each subseries is 10 days and the step is 4 data points. The error bars are 1-sigma above and bellow the estimated value. Choosing a shorter duration of the subseries results in poorly determined values of the amplitudes and phases, while with longer durations, the amplitude and phase variability begins to be artificially smoothed. We restricted this study to the data between JD 2452130 and 2452170, as there is good nightly coverage in this interval.

Surprisingly, in Figure A.5, a phase change is barely seen, but it does not occur when the amplitude reaches a minimum, as we would expect if it were caused by two unresolved close frequencies. The overall shape of the phase changes is well described numerically by two close frequencies of different amplitudes, with the lower amplitude peak also having the lower frequency. Anyway, as stated above, there is a significant shift of $\sim$ 14 days between the expected time of phase change and the observed one.

It remains to determine if the variations observed in V1821 Cygni can be attributed to the interference of more than two close frequencies. In fact, a spectral analysis of the time series of V1821 Cygni cleaned for the low amplitude f_2-f_4($+f_{spurious}$) frequencies (Figure A.6), shows at least two close peaks that are

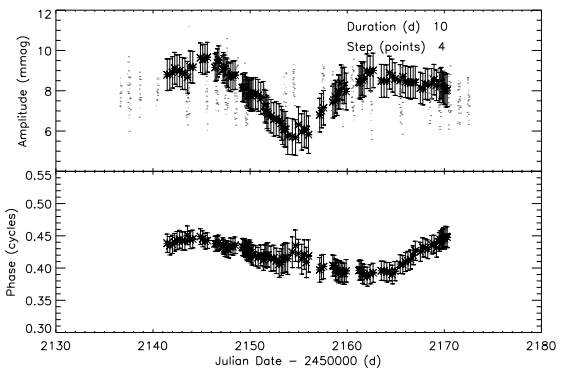

FIGURE A.5— Amplitude and phase variations of V1821 Cygni plotted versus time. Only one part of the whole time series with a good coverage is considered. Each point represents the best fit, in a 10 days length subseries around the point, to the amplitude (top panel) and phase (bottom panel) of a sine with frequency f1. The error bars are 1σ above and bellow the fitted value. The distance between each fitted subserie is 4 data points. The light points are the data points, shifted and scaled for the shake of clarity.

inside the resolution bin of a 10 days time-series (f_5 and f_6 in Figure A.6). As the signal to noise of these peaks is low (3.0 for f_5 and 2.8 for f_6), it is unclear if these are significant close frequencies, just from the spectral analysis. Moreover, there is a detectable daily alias for each of these peaks, with a frequency 1 c/d lower for the frequency close to 8.86 c/d, and in a frequency 1 c/d higher than 8.71 c/d. The single-site nature of these observations and the possible interference among these peaks with f_1 makes it impossible to decide which is the main peak and which the alias. If we restrict the spectral analysis to the part of the time series where the moving fit was applied, the spectrum shows three close frequencies to f_1, at 8.865 c/d (f_5), 8.71 c/d (f_6) and 8.746 c/d (f_7) (Figure A.7). We built an artificial light curve with the frequencies f_1 and these three close frequencies, choosing different values for the phases of the close frequencies. Performing the moving fit analysis to this artificial time series, it is possible to obtain amplitude and phase vs. time diagrams that closely resemble that of V1821 Cygni (Figure A.8). This result does not mean that these three close frequencies are *real* pulsation frequencies of the star. As pointed above, some of them could be aliases of a real frequency, and one (f_7) could be an artifact of the window function, because it is not noticeable in the spectrum of the whole 2001+2002 time series (Figure A.6). Some clarification on the aliasing of the f_5 and f_6 frequencies might be obtained with multi-site

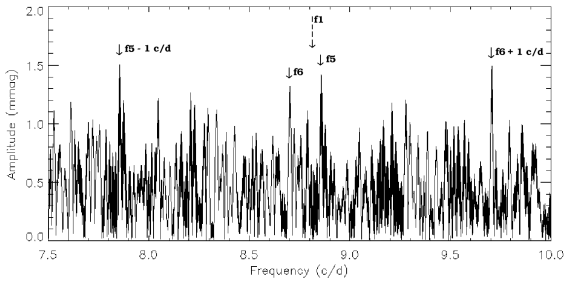

FIGURE A.6— The region around the cleaned frequency f_1, of the time series previously cleaned for the frequencies f_2-f_4.

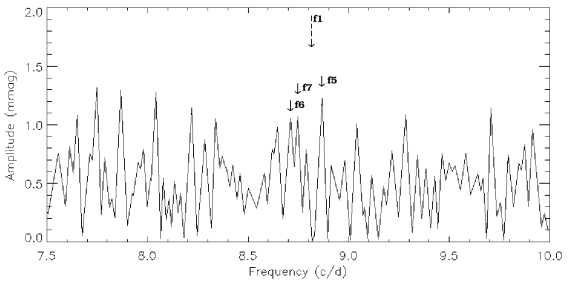

FIGURE A.7— Same as Figure A.6, but for the time interval used in the moving fit analysis.

photometry of this star. The importance of this result resides in that the amplitude and phase variability of the frequency f_1 in V1821 Cygni needs at least two close frequencies to be explained, and indeed it can be reasonably well modelled by three close frequencies that appear in the power spectra. Thus, it is not safe to conclude that the variations of f_1 are intrinsic to the mode, even if a phase change is seen without an apparent connection to an amplitude minimum. In this particular case, both explanations should be kept in mind, until more and/or better data on this star are available.

A.3.5 Comparison with other δ Scuti stars in the field

A similar study was carried for two another δ Scuties in the field, HD 225708 and STARE 8663. The first star has a f_1 frequency with a similar amplitude as V1821 Cygni, and the detected low amplitude frequencies are below a 2 mmag level. Using the same parameters of the *moving fit*, we obtained a behavior compatible with no amplitude nor phase variations (Figure A.9). This result argues once again against an instrumental origin for the phase and amplitude

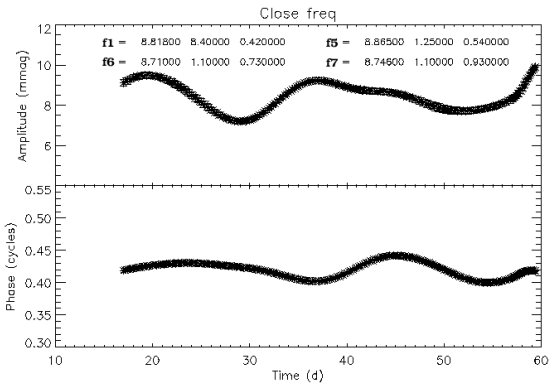

FIGURE A.8— Amplitude and phase variations of a model light curve, built with the three low amplitude peaks distinguishable in Figure A.7, and f_1, and manually adjusting the phases. Note how the first part of the curves resemble Figure A.5.

variations described in the previous sections. For the second star, STARE 8663, in section A.3.3 we had observed amplitude and phase variability of its main pulsation mode. The *moving fit* analysis in this star again shows evidence for this variability (Figure A.10), but in this case a change in phase seems to happen when the amplitude reaches a minimum. This behavior might easily be explained with the presence of two close frequencies in this star.

A.4 Summary and conclusions

A photometric study of the δ Scuti V1821 Cygni, performed with the STARE telescope, has provided a clear detection of 4 frequencies in this star. The frequencies f_1 and f_2 are close to the Zhou, Liu, & Du (2001) values, but f_1 is noticeably lower in our data set, probably due to a faint complex structure of at least two close frequencies to it (f_5 and f_6). We can not discriminate between f_5 and f_6 and their daily aliases due to the single-site nature of our observations. These frequencies close to f_1 can provide an (at least qualitative) explanation for the amplitude and phase variability of f_1. We performed an attempt to discriminate between this explanation and other possible description, namely intrinsical variation of the pulsation mode f_1. This second description could mean that f_5, f_6 and f_7 are just artifacts in the amplitude spectra that appear as a consequence of the variability of f_1, and don't represent real pulsation modes of the star. The moving fit analysis performed showed that we can not reject any of these explanations, even if a change in phase is seen uncorrelated

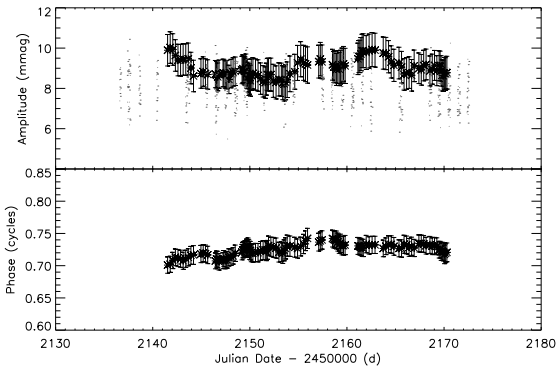

FIGURE A.9— Same as Figure A.5 for another δ Scuti in the same field (HD 225708), showing no evidence for amplitude nor phase variations.

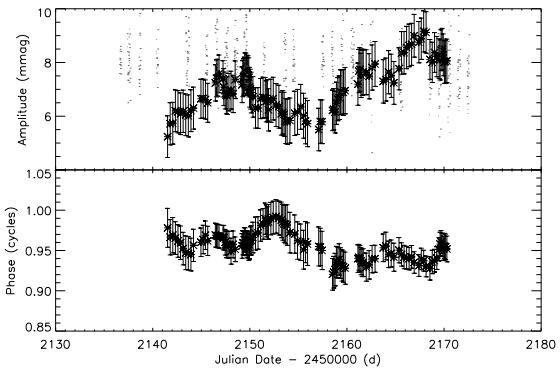

FIGURE A.10— Same as Figure A.5 for another δ Scuti in the same field (STARE 8663), showing evidence for amplitude and phase variations. In this case, a change in phase happens when the amplitude reaches a minimum.

with a minimum in amplitude. This is due to the presence of more than one close frequency. While the frequency f_3 is first reported in this work, the lower amplitude f_4 was detected in the discovery of this star by Delgado et al. (1984), but not seen in the better quality Zhou, Liu, & Du (2001) data set.

Putting all this together, we can conclude that V1821 Cygni suffers significant amplitude and frequency variability on at least two of its pulsation modes (f_1 and f_4).

Finally, an analysis of 5 newly discovered pulsators in the same field of view as V1821 Cygni, resulted in the detection of a δ Sct with at least amplitude variability (STARE 8663). Another one of these pulsators, HD 225708, seems to be consistent with no amplitude nor frequency variations of its highest amplitude pulsation mode, down to the precision of the observations. Frequencies, amplitudes and phases for all these stars have been obtained.