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# Succesful inversion of the stellar statistics equation applied to the Galactic Bulge

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**Abstract.** Since the first decades of this century, attempts have been made to invert the stellar statistics equation. However, problems such as excessive patchiness of extinction in optical star counts or instabilities of an ill-posed problem in the mathematical technique of inversion, hindered the development of the technique. While the extinction problems can be ameliorated by using high quality infrared data, the instabilities must be solved by means of a statistical iterative algorithm of inversion. This contribution shows that Lucy's (1974) algorithm is appropriate to treat this problem and works in some particular cases interesting in Galactic astronomy: for instance, the Galactic bulge. We show that a 3-D density distribution and the luminosity function of the Galactic Bulge can be obtained from high quality infrared star counts without any a priori hypothesis about their functional shape.

Inversion of the stellar statistics equation has been discussed by many authors, although more often this is in theory than in practice, and doubts have been casted on the viability. However, we have found that the problem can be solved in some particular cases: we succesfully got a method which works for the case of the Galactic bulge with high quality near infrared data ($K$ band TMGS survey) at off-plane regions, which have low extinction, by means of Lucy's (1974) algorithm (López-Corredoira et al. 2000 (LHGSM hereafter)).

The stellar statistics equation is a Fredholm integral equation of the first kind (see Trumpler & Weaver 1953; LHGSM). If $N_K(m_K)$ are the measured star counts, the bulge counts, once subtracted a model for the disc, are:

$$N_{K,\mathrm{bulge}}(m_K) = N_K(m_K) - N_{K,\mathrm{disc}}(m_K)$$
$$= \omega \int_0^\infty \Phi_{K,\mathrm{bulge}}(m_K + 5 - 5\log_{10}\rho_K)\Delta_{\mathrm{bulge},K}(\rho_K)\rho_K^2 d\rho_K, \qquad (1)$$

where $\Delta$ is the unknown function and $\Phi$ is the kernel of the integral equation. $\Phi_{K,\mathrm{bulge}}(M) = \int_{-\infty}^M \phi_{K,\mathrm{bulge}}(M)dM$, $\phi$ is the normalised luminosity function. We make the assumption that the bulge luminosity function is independent of position. $\rho_K = 10^{0.2a_K(r)}r$ and $\Delta_K = D(r)\frac{r^2dr}{\rho_K^2 d\rho_K}$ $D$ is the density of stars and $r$ the distance. When $\Phi$ is the unknown function instead of $\Delta$, then a new change of variable can be made: $M_K = m_K + 5 - 5\log_{10}\rho_K$, and a new Fredholm equation of the first kind is obtained:

$$N_K(m_K) = 200(\ln 10)10^{\frac{3m_K}{5}} \int_{-\infty}^\infty \Delta_K(10^{\frac{5+m_K-M_K}{5}})10^{\frac{-3M_K}{5}}\Phi_K(M_K)dM_K. \quad (2)$$

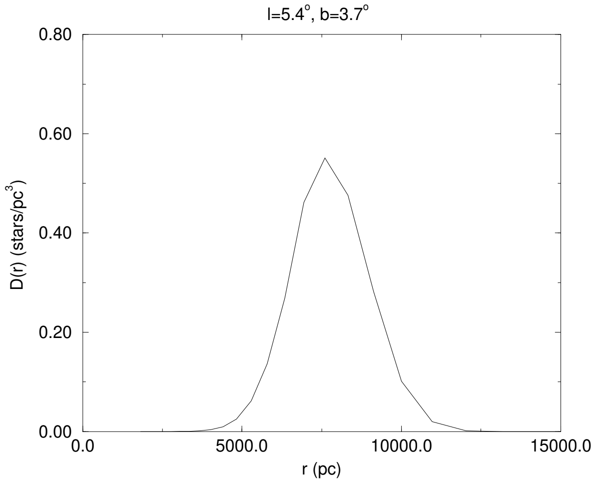


Figure 1. Density along the line of sight ($l = 5.4°$, $b = 3.7°$).

In this case, the kernel is $\Delta$ instead of $\Phi$. Any method of inverting eq. (1) is also applicable to this integral equation (2).

The inversion of integral equations such as the above equations is ill-conditioned. Typical analytical methods for solving these equations (see Balázs 1995) cannot achieve a good solution. Since the functions in these equations have a stochastic rather than analytical interpretation, it is to be expected that statistical inversion algorithms will be more robust. From among these statistical methods, we have selected Lucy's (1974) algorithm, an iterative method, the key to which is the interpretation of the kernel as a conditioned probability and the application of Bayes' theorem.

By means of an iterative method of inverting (1) and (2) (see LHGSM) the structure and the luminosity function of the Galactic bulge can be derived. In Fig. 1, it is shown the density along a particular line of sight derived from the inversion. By using this method applied to the different lines of sight, we have obtained that the solution for the 3-D structure of the bulge is approximately a triaxial ellipsoid (see LHGSM or the poster "Is the Milky Way a Double-barred Galaxy?" by Hammersley et al. in this conference).

## References

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