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Physics Letters B

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# Aligned electromagnetic excitations of a black hole and their impact on its quantum horizon

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## ARTICLE INFO

### Article history:

Received 20 June 2008

Received in revised form 13 September 2008

Accepted 10 December 2008

Available online xxx

Editor: M. Cvetič

### PACS:

42.50.Lc

03.70.+k

11.10.Ef

## ABSTRACT

We show that elementary aligned electromagnetic excitations of black holes, as coming from exact Kerr–Schild solutions, represent light-like beams pulses which have a very strong back reaction on the metric and change the topology of the horizon. Based on York's proposal, that elementary deformations of the BH horizon are related with elementary vacuum fluctuations, we analyze deformations of the horizon caused by the beam-like vacuum fluctuations and obtain a very specific feature of the topological deformations of the horizon. In particular, we show how the beams pierce the horizon, forming a multitude of micro-holes in it. A conjecture is taken into consideration, that these specific excitations are connected with the conformal-analytic properties of the Kerr geometry and are at the base of the emission mechanism.

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## 1. Introduction

The interest in Hawking radiation from a black hole [1–7] is not decreasing. New derivations of this law, some of them very different from the original one and opening new avenues in quantum physics, have appeared in the last years. Even more interesting, from the experimental (i.e., observational) side results are quickly reaching limits which clearly indicate that some specific properties of black holes will be testable in the near future.

There are a few semiclassical explanations of the origin of the quantum evaporation process of BHs. Some of the most popular are: (i) Evaporation is related with 'quasi-normal' modes of black hole excitations [8,9] and the emitted particles are radiated by means of a quantum tunnelling process from under the horizon [9]. (ii) The loop quantum gravity approach [10,11] proclaims the quantum horizon, experiencing topological fluctuations, to be itself the source of the quantum radiation. (iii) Strong fields near the horizon lead to virtual pair creation in its outer vicinity, and while one of the particles falls into the horizon the other keeps going to infinity [9,12,13]. This process can also be considered as a quantum tunneling via a classically energetically prohibited region, the Klein paradox [12].

In the original derivation of BH evaporation, Hawking described thermal radiation as quantum tunneling triggered by vacuum fluctuations, however, the treatment was based upon a fixed background, without a back reaction causing the fluctuations of the background geometry. Consideration of the back reaction of vacuum fluctuations and their impact on the horizon was initiated by York [14] who used spherical zero-point fluctuations (ringing modes) and the corresponding deformation of the space–time metric. This dynamical approach was also supported by loop quantum gravity [6] and in subsequent work [5,7] (see also the references in [5]). However, the treatment was restricted to simplest deformations of the horizon, caused by spherical or ellipsoidal quasi-normal modes. Meanwhile, recent analysis of rotating black holes [15] in the Kerr–Schild formalism showed that the *exact* electromagnetic excitations on the Kerr background has to be *aligned with the Kerr congruence*. As a consequence, the elementary electromagnetic excitations turn out to be very far from the usual spherical harmonics. They constitute a brand new basis for elementary excitations and for elementary deformations of the horizon.

The present treatment, based on these exact solutions, shows that horizons acquire a very specific topological deformation subjected to influence of vacuum electromagnetic excitations. We show that an elementary aligned excitation is accompanied by the formation of a singular light-like beam, which emerges from the black hole. The influence of those beams on the horizon was analyzed in our previous paper [15]. It was shown there that the horizon is very sensitive to such beams, which form thereby nar-

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row holes connecting the inner and outer regions. Even weak elementary beams may be able to break through the Kerr black hole horizon. Moreover, the elementary deformations of the horizon turn out to be of topological nature. We consider also the wave aligned electromagnetic excitations of the Kerr solution and show that they are asymptotically exact in the low-frequency limit. This result allows us to use aligned wave excitations for modeling excitations of massive black holes through weak fields of virtual photons, and to arrive to a new feature of deformation of the horizon, by invoking a large set of microscopic holes in the horizon, caused by singular beams.

We put forward the conjecture that the complex-analytic structure of the Kerr congruence and of the wave excitations which are aligned with the Kerr congruence are at the base of the emission mechanism.

## 2. Aligned electromagnetic solutions in the Kerr–Schild geometry

As is well known [15–17], the Kerr–Newman metric is the unique regular one representing a much broader Kerr–Schild class of Einstein–Maxwell solutions. There is an infinite set of exact rotating Kerr–Schild solutions containing semi-infinite light-like beams (the singular  $pp$ -waves in asymptotic limit, see e.g. [18]). The electromagnetic field of these solutions is aligned with the principal null congruence of the Kerr geometry. We show in this Letter that correspondingly aligned wave excitations of the Kerr geometry tend to the above mentioned exact stationary Kerr–Schild solutions in the low-frequency limit. Below, we summarize the discussion and specify the whole argument for the case of the Kerr–Newman solution in the Kerr–Schild form.

The peculiarities of the Kerr–Schild approach rely on the Kerr–Schild ansatz for the metric, which is adjusted to an auxiliary Minkowski space–time with a metric  $\eta_{\mu\nu}$ , contrary to other approaches, in which the coordinate system is adjusted to a position of the horizon which is unstable itself and, as we will show, is very sensitive to the electromagnetic field. The Kerr–Schild approach allows one to see some peculiarities of the solutions which are not seen in other approaches. In particular, it exhibits complex-analytic properties of the black hole geometries which are related with two sheets of the space–time. On the Kerr–Schild background one sees that the Kerr geometry is two-sheeted, having an ‘in’ (–) and an ‘out’ (+) sheet, and corresponding  $g^+(x)$  and  $g^-(x)$  metrics:

$$g_{\mu}^{(\pm)} v = \eta_{\mu\nu} v - 2Hk_{\mu}^{(\pm)} k_{\nu}^{(\pm)}, \quad (1)$$

which are different on the ‘in’ and on the ‘out’ sheets. Moreover, the fields on the ‘in’ and ‘out’ sheets do not interact with each other. In particular the principal null directions  $k_{\nu}^{(\pm)}(x)$  are different on different sheets, being determined by different functions  $Y^+(x)$  and  $Y^-(x)$ , in the null Cartesian coordinates  $u = z - t$ ,  $v = z + t$ ,  $\zeta = x + iy$ ,  $\bar{\zeta} = x - iy$ , namely

$$k_{\mu}^{(\pm)} dx^{\mu} = P^{-1}(du + \bar{Y}^{\pm} d\zeta + Y^{\pm} d\bar{\zeta} - Y^{\pm} \bar{Y}^{(\pm)} dv). \quad (2)$$

The known Kerr singular ring is a branch line of space, and forms a gate to the negative sheet of the metric. The Kerr principal null congruence (PNC) propagates from the ‘in’ sheet of the Kerr space to the ‘out’ sheet through the Kerr singular ring, covering the auxiliary Minkowski space–time twice. This peculiarity of space–time has not been observed earlier by analysis of the usual spherical black hole solutions and leads to far-reaching consequences, as we intend to show.

The electromagnetic field corresponding to the exact Kerr–Schild solution has to be *aligned* with the Kerr principal null congruence. This means that the Maxwell tensor of strength  $F_{\mu}^{\nu}$  and vector potential  $A_{\mu}$  satisfy the following alignment condition

$$k^{\mu} F_{\mu}^{\nu} = f(x)k^{\nu}, \quad k^{\mu} A_{\mu} = 0. \quad (3)$$

Now, since the vector fields  $k_{\mu}^{\pm}$  are different on the ‘in’ and ‘out’ sheets of the Kerr geometry, one sees that the ‘in’ and the ‘out’ aligned fields cannot possibly be on the same sheet simultaneously. This is in accordance with the structure of the algebraically special Kerr–Schild solutions. Therefore, in the Kerr geometry the ‘in’ and the ‘out’ modes of the aligned electromagnetic waves are positioned on the two different sheets of the real Kerr space and do not interact (at least classically). The alignment condition is very strong and leads to important physical consequences. It resembles the condition for exclusion of the longitudinal modes in dual-string modes and is the origin of the complex-analytic properties of the Kerr geometry.<sup>1</sup>

Note that, in the ordinary perturbative approach based on quasi-normal modes, the important two-sheetedness of the Kerr geometry has been ignored. The perturbative approach breaks analyticity of the solutions and leads to a drastic change in the structure of the usual quasi-normal and aligned electromagnetic excitations of (rotating) black holes. In particular, the analytic aligned wave excitations lead to the formation of light-like singular beams emanating from the black hole and changing the properties of its horizon, breaking up the usual classical structure of the black hole.

We show here that the aligned wave solutions are asymptotically exact for slowly varying excitations, that is, in the low-frequency limit. Similar beams will also appear for the aligned excitations of the rotating sources without horizons considered in [15,18].

## 3. Exact Kerr–Schild solutions with singular beams

For the readers’ convenience we recall here the structure of the exact Kerr–Schild solution displaying the aligned electromagnetic field which produces the singular beams [17]. The exact electromagnetic field which is aligned with the Kerr null congruence,  $k^{\mu}$ , depends on an arbitrary holomorphic function  $\psi(Y)$ , which determines the Kerr–Schild (KS) metric (1) via the function

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta}. \quad (4)$$

In the Kerr–Newman solution,  $\psi = q$  is the charge. However, any holomorphic function  $\psi(Y)$  yields also an exact aligned solution. The function

$$Y = e^{i\phi} \tan \frac{\theta}{2} \quad (5)$$

is a projective angular coordinate. It determines the Kerr congruence (2) and also the KS tetrad  $e^a$  with the real directions

$$e^3 = Pk_{\mu} dx, \quad e^4 = dv + he^3, \quad h = HP^{-2}, \quad (6)$$

and the mutually complex conjugated forms

$$e^1 = d\zeta - Y dv, \quad e^2 = d\bar{\zeta} - \bar{Y} dv. \quad (7)$$

For the Kerr congruence satisfying the *geodesic* and *shear-free* conditions:

$$Y_{,2} = Y_{,4} = 0, \quad (8)$$

the Einstein–Maxwell field equations were integrated by Debney, Kerr and Schild [17] in a general form. They reduced them to a system of two equations for the electromagnetic field and two more for the gravitational field. The electromagnetic field aligned with the Kerr congruence admits the self-dual tetrad components,

$$\mathcal{F}_{12} = AZ^2, \quad \mathcal{F}_{31} = \gamma Z - (AZ)_{,1}, \quad (9)$$

<sup>1</sup> Another source of complex analyticity is the Kerr theorem which determines the Kerr congruence,  $k^{\mu}(x)$ , in terms of complex-analytic surface in twistor space [16,21].

where  $\mathcal{F}_{ab} = e_a^\mu e_b^\nu \mathcal{F}_{\mu\nu}$ , and the function  $Z$  is a complex expansion of the congruence, which for the stationary Kerr–Newman solution is inversely proportional to a complex radial distance  $Z = -P/(r + ia \cos \theta)$ , where  $P = 2^{-1/2}(1 + Y\bar{Y})$ . Functions  $A$  and  $\gamma$  obey Eqs. (B.1), (B.2), given in Appendix B. Explicit solutions were obtained in [17] for the case stationary fields without wave excitations,<sup>2</sup> which corresponds to  $\gamma = 0$ , and

$$A = \psi/P^2, \quad \psi_{,2} = \psi_{,4} = 0. \quad (10)$$

The function  $\psi(Y)$  is analytic and can be represented as an infinite Laurent series:  $\psi(Y) = \sum_{n=-\infty}^{\infty} q_n Y^n$ .

If the function  $\psi(Y)$ ,  $Y \in S^2$ , is not a constant, it has to contain at least one pole which may also be at  $Y = \infty$  (or  $\theta = \pi$ ). So, except for the Kerr–Newman solution, for which  $\psi(Y) = q = \text{const}$ , the solutions  $\psi(Y)$  need to be singular at some angular directions  $Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}$ . We then consider the ansatz:

$$\psi(Y) = \sum_i \frac{q_i}{Y - Y_i}, \quad A = \psi(Y)/P^2. \quad (11)$$

Singular electromagnetic fields have a strong back reaction on the metric, via the function  $\psi(Y)$  in (4). For  $q_i = \text{const}$ , these functions yield exact, self-consistent solutions of the full system of Kerr–Schild equations.

In this case, the term  $\mathcal{F}_{31}$ , which describes the null em-radiation  $\mathcal{F}_{\mu\nu} = \mathcal{F}_{31} e_\mu^3 e_\nu^1$  along the Kerr congruence, takes the form

$$\mathcal{F}_{31} = (AZ)_{,1} = \sum_i \left\{ - (Z/P)^2 \frac{q_i}{(Y - Y_i)^2} - 2(Z/P)^2 \frac{q_i P_Y}{P(Y - Y_i)} + 2ia\bar{Y} \frac{q_i}{Y - Y_i} P^{-2} (Z/P)^3 \right\}. \quad (12)$$

Since  $Z/P = 1/(r + ia \cos \theta)$ , this expression is singular at the Kerr ring and falls off as  $r^{-2}$  and  $r^{-3}$  with the distance. The first term in the sum is leading. It contains the poles  $\sim \frac{q_i}{(Y - Y_i)^2}$  which are singular along the lines of congruence in the direction  $Y_i$  and do not fall off, taking asymptotically the form of singular  $pp$ -wave Peres solutions [21] describing the narrow singular electromagnetic beams in each of the angular directions  $Y_i$  [18], i.e. semi-infinite ‘axial’ singular lines which destroy the horizon and which lead to the formation of topological holes in it.

The properties of the horizons of these solutions were considered in [15]. It was shown that the black hole horizon is pierced by the axial singularity, with the appearance of a tube-like region which connects the internal with the external region, thus possibly allowing matter to escape from the BH. As a result, the Kerr singularity turns out to be ‘half-dressed’. The structure of the horizons for the solutions containing axial singular lines follows from the Kerr–Schild form of the metric for the function  $H = \{mr - \psi(Y)^2/2\}/(r^2 + a^2 \cos^2 \theta)$ , where the oblate coordinates  $r, \theta$  are used on the flat Minkowski background  $\eta^{\mu\nu}$ . For the rotating solutions, the horizon splits into four surfaces. Two of them correspond to the static limit,  $r_{s+}$  and  $r_{s-}$ , determined by the condition  $g_{00} = 0$ , and the other two surfaces correspond to the event and Cauchy horizons,  $S(\chi^\mu) = \text{const}$ , which are the null surfaces determined by the condition  $g^{\mu\nu}(\partial_\mu S)(\partial_\nu S) = 0$ .

The simplest axial singularity is the pole  $\psi = q/Y$ . In this case, the boundaries of the ergosphere,  $r_{s+}$  and  $r_{s-}$ , are determined by  $g_{00} = 0$  and the solution acquires a new feature: the surfaces  $r_{s+}$  and  $r_{s-}$  are joined by a tube, conforming a simply connected surface. The surfaces of the event horizons are null ones and obey the differential equation  $(\partial_r S)^2 \{r^2 + a^2 + (q/\tan \frac{\theta}{2})^2\} - 2Mr\} -$

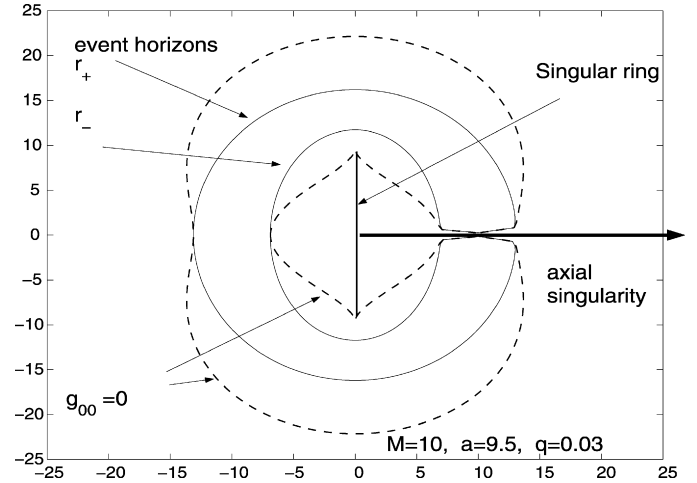


Fig. 1. Hole in the horizon of a generic rotating black hole formed by a singular beam directed along axis of symmetry.

$(\partial_\theta S)^2 = 0$ . The resulting structure for the horizon is illustrated in Fig. 1 (taken from [15]).

The two event horizons are joined into one connected surface, and the surface of the event horizon lies inside the boundary of the ergosphere. As a consequence, the axial singularities lead to the formation of holes in the BH horizon, thus opening the interior of the BH to external space.

#### 4. Aligned wave excitations and beam-like pulses

Corresponding exact solutions with wave electromagnetic excitations on the Kerr background have been obtained and investigated in [15,18]. They are asymptotically exact in the low-frequency limit, i.e. are also self-consistent, taking into account back reaction. In the far zone the wave beams were considered in [18] and it was shown that locally they have the structure of the self-consistent exact  $pp$ -wave (Peres) solutions. Note, that similar singular ‘axial’ strings were obtained long ago in the Robinson–Trautman class of solutions [21], and also for fermionic wave excitations on the Kerr geometry [19].

Wave excitations propagating in the direction  $Y_i$  are described by means of the function

$$\psi_i(Y, \tau) = q_i(\tau) \exp\{i\omega\tau\} \frac{1}{Y - Y_i}, \quad (13)$$

where  $q_i(\tau)$  is a slowly varying amplitude of excitation with analytic dependence on the retarded time  $\tau$ . Similar to the usual treatment of the plane waves, such infinitely extended beam solutions have to be replaced by physically realizable solutions with finite extension. In particular, the wave and non-stationary generalizations of these solutions have to contain some time-dependent amplitude factor  $q = q(\tau)$  which turns them into beam-like pulses. The  $pp$ -wave singularity can also be regularized, as it was considered in [18].

The equations for the gravitational sector, obtained by Debney, Kerr and Schild (DKS), are given in Appendix A. The most essential fact in the treatment of these wave and non-stationary Kerr–Schild solutions is the appearance of an extra radiation field  $\gamma$ , which is absent in the term  $H$  (4) and does not have immediate influence on the metric tensor. However, it leads to the appearance of null radiation  $\mathcal{F}_{\mu\nu} = \gamma Z e_\mu^3 e_\nu^1$  which is described by the Vaidya shining star solution [21]. This radiation leads to the loss of mass of the black hole, and we show in Appendix A that it is small for low-frequency solutions, since the rhs of the gravitational equations  $\gamma \sim \psi$  tends to zero in the low-frequency limit  $\omega \rightarrow 0$ . Therefore,

<sup>2</sup> About this problem see also [20].

the wave beam-like pulses tend to self-consistent Kerr–Schild solutions in this limit.

The structure of the solutions contains the following very important physical peculiarity: the smallness of the function  $\gamma$  does not mean that the function  $\psi(Y, \tau)$  in (4) is required to be small too. The electromagnetic field determined by  $A = \psi/P^2$  and the corresponding distortion of the metric and horizon by the function  $H$  are independent from the frequency and can be in fact very strong. Note also, that there is one subtle point in our treatment related with the fact that, due to the poles in the angular directions  $Y_i$ , the limit  $\gamma \rightarrow 0$  as  $\omega \rightarrow 0$  is not a uniform one, which demands a thorough treatment. However, the general integral expressions for  $\gamma$  contain a free term, and in Appendix B we show that this term may be used for the regularization of these poles, without any influence on the function  $A$ .

### 5. Aligned wave excitations and micro-holes in the horizon

Since the aligned solutions here considered tend to the exact ones in the asymptotic limit  $\gamma \rightarrow 0$ , this also means, physically, that they must tend to the exact solution corresponding to ‘ringing modes’ of a black hole subjected to a weak external electromagnetic excitation. In particular, it may be the vacuum field of virtual photons. It could be reduced to a standard quantum treatment of scattering plane waves on a potential barrier, if only the extra Kerr–Schild demand of the alignment of the exact solutions with the Kerr congruence (3) would be obeyed. However, this demand is very hard, and the plane waves cannot reside in the Kerr background. Even in the limit of zero mass term, the Kerr background is not Minkowski one and has a two-sheeted topology which does not admit plane waves. This situation is close related with the twistor analytic structure of the Kerr geometry, and prevents the use of a standard scattering approach. The character of this paper does not allow us to discuss this problem.<sup>3</sup> Because of that, we will follow the simple idea of York [14] to consider the back-reaction of an elementary excitation of a black-hole caused by an elementary interaction with a virtual photon. In most of the previous treatments of this sort, deformation of the horizon was related with the replacements  $m \rightarrow m + \delta m$  and  $J \rightarrow J + \delta J$ , retaining the spherical topology of the horizon. In our case, in accordance with the discussed solutions, an elementary excitation aligned with the Kerr congruence represents a light-like beam, which in the far zone takes the form of  $pp$ -wave string-like solution, and the corresponding elementary deformation of the horizon changes its topology, even at the low-frequency limit.

We neglect the recoil and use expressions (A.4) for the stationary case, which allow us to get the exact solution for function  $A$  in the form containing the wave excitations

$$A = \psi(Y, \tau)/P^2. \quad (14)$$

This equation is linear in  $\psi$  and the total excitation caused by the virtual photons will be a sum over elementary excitations in the distinct directions  $Y_i$ ,

$$\psi(Y, \tau) = \sum_i \frac{q_i(\tau)}{Y - Y_i} \exp\{i\omega_i \tau\}. \quad (15)$$

The wave solution (15) with many excitations will become exact on the Kerr background, while the back reaction will break self-consistency, leading to some disclosure in the gravitational sector, as it is discussed in Appendix B, this disclosure is proportional to

the term  $A\bar{\gamma}$  in Eqs. (A.5), and has to tend to zero in the low-frequency limit together with  $\gamma \rightarrow 0$ . The corresponding retarded time has the form [18]  $\tau = t - r - ia \cos \theta$ , which allows us to obtain the general retarded-time solution  $\gamma = \gamma_0 + \gamma_f$  as the sum of the partial solution  $\gamma_0$  containing series of poles,

$$\gamma_0 = \sum_i c_i(\tau) \frac{1}{P^2 Y (Y - Y_i)} \quad (16)$$

with oscillating factors

$$c_i(\tau) = i\omega 2^{1/2} q_i(\tau) \exp\{i\omega_i \tau\}, \quad (17)$$

and the term

$$\gamma_f = \frac{\phi(Y, \tau)}{P} \quad (18)$$

which is determined by a free function  $\phi(Y, \tau)$ .

The free term  $\gamma_f$  has to be tuned to have the same series of poles and the same oscillating factors of opposite sign to provide regularization of poles by subtraction. It is shown in Appendix B that such a regularization does not touch the function  $A$  and provides a uniform low-frequency limit  $\gamma_{\text{out}} \rightarrow 0$ . Following to analogues with family of Green functions one can introduce the step function  $\theta(r) = \pm 1$ , separating the solutions on the positive sheet  $r > 0$  from the solutions on the negative one,  $r < 0$ , and consider the ingoing field of virtual photons as a non-regularized one,  $\gamma_{\text{in}} = \gamma_0$ , while the field  $A$  and the regularized field  $\gamma_{\text{reg}} = \gamma_{\text{in}} - \gamma_{\text{comp}}$  may be considered as outgoing radiation  $\gamma_{\text{out}} = \gamma_{\text{reg}}$ .

The question can be asked, what is the mechanism which provides such a compensating regularization and discontinuity of the solutions? We do not know a precise answer, although it is quite clear that the discontinuity has to be related to some source<sup>4</sup> positioned in the vicinity of the Kerr disk  $r = 0$ , and that regularization minimizes the interaction between the vacuum field  $\gamma_{\text{in}}$  and the one induced by the Kerr source excitations, in the form of the field  $A$ . The process of scattering may be interpreted in this case as a type of scattering of the incoming vacuum radiation on the stringy structure of the Kerr geometry.

It should be emphasized once more that there is a principal difference between the roles played by the fields  $A$  and  $\gamma$ . The fields  $\gamma$  as well as  $\gamma_{\text{reg}}$  do not act immediately on the metric, being respectable ones for the radiation and the appearance of singular micro-beams covering the celestial sphere  $S^2$ . Its influence on the metric is indirect and occurs only via the loss of mass by radiation, in accordance with expression (A.6), where  $\gamma_{\text{reg}}$  has to be substituted by the outgoing radiation,  $\gamma_{\text{out}} = \gamma_{\text{reg}}$ . At the same time, the exact solutions for the electromagnetic field  $A$  on the Kerr–Schild background, have direct influence on the metric, leading to the formation of the holes at the horizon. The direct influence of the field  $A$  on the metric and on the horizon will result into formation of multiple micro-holes in the horizon and its topological fluctuations, as depicted in Fig. 2.

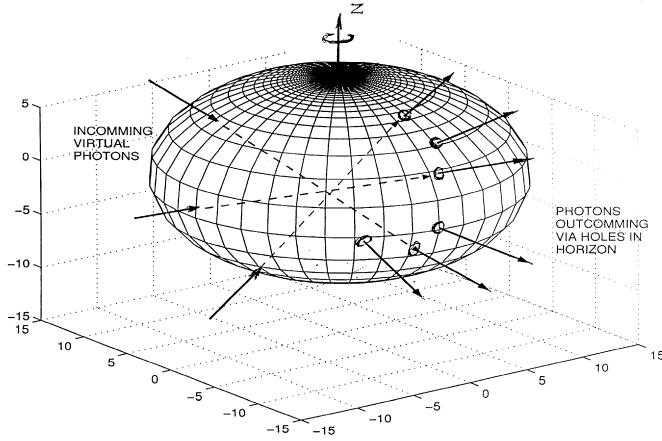
In the far zone the stress tensor of the electromagnetic field is determined by the leading term  $\mathcal{F}_{31}$ , (9), which describes the null electromagnetic radiation along the rays of the congruence  $e_{\mu}^3 = Pk_{\mu}$ . At the same time, the stress–energy tensor is represented by the  $T_{33}$  component, which is determined by the  $\gamma$ -term

$$T_{\mu\nu} = T_{33} e_{\mu}^3 e_{\nu}^3 = \frac{1}{2} \gamma_{\text{out}} \bar{\gamma}_{\text{out}} P^2 k_{\mu} k_{\nu}. \quad (19)$$

Radiation propagates along the Kerr congruence which is geodesic with the null tangent vector  $k^{\mu}$ . This means that radiation is massless. Each elementary  $i$ th excitation, with a pole  $\sim \frac{1}{(Y - Y_i)}$ , propagates along the  $i$ th null ray in the direction  $k^{\mu}(Y_i)$ . The in-going

<sup>3</sup> The replacement of plane waves by twistor null lines (beams) was shown by Witten in the new approach to scattering amplitudes in twistor space [22], inspired by pioneering work of Nair [23].

<sup>4</sup> The problem of the Kerr source is one of the most hard and ambiguous. There are evidences of a stringy structure of the Kerr source [24,25,33].



**Fig. 2.** Excitation of a black hole by the zero-point field of virtual photons forming a set of micro-holes at its horizon.

radiation propagates from  $r = -\infty$  on the negative sheet of the metric towards the Kerr disc  $r = 0$ . The outgoing radiation propagates on the positive sheet in the direction  $r \rightarrow \infty$ . The averaged term on the rhs of the gravitational equation (A.6),  $\langle \gamma_{\text{out}} \bar{\gamma}_{\text{out}} \rangle = \sum_i \langle \gamma_i \text{out} \bar{\gamma}_i \text{out} \rangle$  is positive and determines the incoherent radiation corresponding to the known Vaidya shining-star solution [14,21].

## 6. Discussion

In a number of recent works, the origin of thermal emission from black holes has been related with conformal-analytical structures inherent to two-dimensional,  $(1+1)$ -dimensional and  $(2+1)$ -dimensional black holes [27–32]. The two-dimensional case is the most familiar and the corresponding conformal field theories have been elaborated in detail, including corresponding quantum gravity issues. Quantum effects of the low-dimensional black holes are related to some Virasoro algebra which, on the other side, represents the core of the (super)string theory. Therefore, one observes the growing tendency that the methods of superstring theory acquire increasing meaning in the physics of black holes. The Kerr–Schild structure of the Kerr geometry is based on the Kerr theorem, the complex-analytic structure of which is expressed in terms of twistors. Twistorial analyticity of the Kerr geometry [33] is a *natural* four-dimensional analog to two-dimensional conformal-analytical stringy structures. The analyticity of electromagnetic excitations of the Kerr geometry is expressed in terms of the alignment of the electromagnetic fields with the holomorphic structure of the Kerr congruence, which is determined by the Kerr theorem in terms of an analytic surface in twistor space [16,33].

A central role in the twistorial structure is played by the two-dimensional sphere  $S^2$ , parametrized by the analytic function  $Y(x)$  which is a projective angular coordinate. The singular Kerr–Schild solutions are described by holomorphic fields on this sphere, just similar to topological excitations of the WZW model. Therefore, the mechanism of the origin of the quantum spectrum in the Kerr–Schild geometry is to be similar to the suggested one for 2- and  $(2+1)$ -dimensional BTZ black holes, which is also based on the topological WZW model [29,30]. In particular, it was obtained in [26] that a BTZ black hole may be represented as a section of the four-dimensional Kerr black hole in the equatorial plane  $\theta = \pi/2$ .<sup>5</sup>

<sup>5</sup> At first sight this looks strange, since a BTZ black hole has a negative scalar curvature, contrary to the zero value it has in the Kerr solution. However, there is a very simple argument in favor of this statement. Indeed, separating the  $\theta$  variable

It is known that the dual string models [34] are based on conformal field theory [34,35,38], the core of which is formed by quantum oscillators obeying a Virasoro algebra [34]

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}, \quad (20)$$

with similar expression for conjugate generators  $\bar{L}_n$ . The central charge  $c$  depends on the details of the quantum system considered and was studied for a large variety of different stringy models [34]. However, numerous investigations of different models have shown [30] that details of the theories are not important, and that the key requirement is the existence of the conformal structure, in particular, the  $S^1$  isometry providing the algebra of the gauge field transformations  $\text{Diff } S^1$ . The Bekenstein–Hawking thermal spectrum is reached in most of these models on the base of Cardy’s formula [32], with independence of the value of the central charge [27,30]. Similarly, the conformal group of the Kerr–Schild geometry forming the core of its analytic properties is related with the projective angular coordinate  $Y(x)$ , which is one of the three twistor coordinates of the projective twistor space. It has the stringy structure [36,37] and may determine the spectrum of the radiation from the Kerr black hole similar to BTZ black holes in accordance with Cardy’s formula. This demands extra considerations.

The main goal of this Letter was to show the existence of a very nontrivial beam-like form of excitations and of a related radiation of the Kerr–Schild geometry, as well as a quite nontrivial form of a corresponding topological deformation of the horizon, which are based on analytic properties of the Kerr–Schild geometry and determined by the alignment conditions for the electromagnetic excitations. In conclusion, we touched upon the old question: where is the black hole radiation created? Contrary to the familiar point of view that radiation is created near the horizon, we arrive at the conclusion that the horizon plays a rather passive role. Being perforated by a series of beams, it provides a way to escape for radiation, while the creation of the radiation itself is related with the gravitational anomaly which occurs under regularization of the poles of the energy–momentum tensor. In this respect we share the point of view discussed in [3]. Indeed, in the case of a stationary field  $A$ , i.e.  $\dot{A} = 0$ , the field  $\gamma$  is decoupled, the free flow of zero-point radiation  $T_{(\gamma)}^{\mu\nu} = \frac{1}{2}\gamma\bar{\gamma}P^2k^\mu k^\nu$  obeys the conservation law  $\partial_\mu T_{(\gamma)}^{\mu\nu} = 0$ , and the field  $\gamma$  may be decoupled from this system of equations [18,25]. Creation of the wave field  $A$  is similar to a scattering process which, in accordance with [3], leads to an anomaly in general covariance, taking the form of a non-conservation of the energy–momentum tensor and signaling towards the interaction with some source in the core of the black hole, which is apparently the true source of radiation.

This conjecture demands extra investigations of the stringy Kerr source at the quantum level, which will be the object of further work and is beyond the scope of this Letter.

## Acknowledgements

We thank Valery Frolov and Theo Nieuwenhuizen for useful discussions on the earlier stage of this work. This work was supported by grant RFBR 07-08-00234, by MEC (Spain), BFM2006-02842, PR2006-0145, and by AGAUR, grant 2007BE-100003 and contract 2005SGR-00790.

in the Kerr solution, one has to remember that it is an angular variable giving a positive contribution to the total scalar curvature of the Kerr space–time, which is zero. Consequently, the scalar curvature  $R_{2+1}$  of the  $2+1$  factor-space will be negative, in full correspondence with the BTZ black hole.

## Appendix A

We consider here the equations for the gravitational sector obtained by Debney, Kerr and Schild (DKS)

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}M = A\bar{\gamma}\bar{Z}, \quad (\text{A.1})$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad M_{,4} = 0 \quad (\text{A.2})$$

where  $\mathcal{D}$  is given by

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2. \quad (\text{A.3})$$

We assume that the mass of the black hole  $m$  is much bigger than the energy of the excitations, neglect the possible recoil, and use the expression for the stationary function  $Y(x)$  and its tetrad derivatives. This yields

$$P = 2^{-1/2}(1 + Y\bar{Y}), \quad Y_{,3} = -\bar{Z}P_{\bar{Y}}/P, \quad (\text{A.4})$$

and allows one to reduce Eqs. (A.1) and (A.2) to the very simple form

$$m_{,\bar{Y}} = A\bar{\gamma}P^3, \quad A = \sum_i \psi_i/P^2, \quad (\text{A.5})$$

$$\dot{m} = -\frac{1}{2}P^4\gamma\bar{\gamma}. \quad (\text{A.6})$$

It is known [21] that the last equation determines the loss of mass by radiation. The right sides of both gravitational equations will be small for the low-frequency aligned wave excitations, since the function  $\gamma$  will be of the order  $\gamma \sim \dot{\psi} \sim i\omega\psi$ .

## Appendix B

We consider here in more detail the Kerr–Schild equations obtained in [17] for the electromagnetic sector

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}A = 0, \quad A_{,4} = 0, \quad (\text{B.1})$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0, \quad \gamma_{,4} = 0, \quad (\text{B.2})$$

where  $\mathcal{D}$  is given by (A.3). We neglect the recoil and use expressions (A.4) for the stationary case, what allows us to get the exact solution for the function  $A$  in a form containing the wave excitations

$$A = \psi(Y, \tau)/P^2, \quad (\text{B.3})$$

and reduce Eq. (B.2) for the adjoined field  $\gamma$  to the very simple form

$$\dot{A} = -(\gamma P)_{,\bar{Y}}. \quad (\text{B.4})$$

These equations are linear in  $\psi$  and  $\gamma$ , and the total excitation caused by the virtual photons will be a sum over elementary excitations in the distinct directions  $Y_i$ ,  $\psi(Y, \tau) = \sum_i \frac{q_i(\tau)}{Y - Y_i} \exp\{i\omega_i\tau\}$ .

The wave solution (15) with many excitations will be exact on the Kerr background, while the back reaction will break self-consistency, leading to some disclosure in the gravitational sector. As it was discussed in the previous section, this disclosure is proportional to the term  $A\bar{\gamma}$  in Eqs. (A.5), and has to tend to zero in the low-frequency limit together with  $\gamma \rightarrow 0$ . However, the limit  $\gamma \rightarrow 0$  will not be uniform because of the poles, and a thorough procedure is necessary for the ‘regularization’ of singularities.

Eq. (B.4) can be integrated in the general form [18]. To perform it, one should introduce the retarded-time parameter  $\tau$

obeying the constraints<sup>6</sup> satisfy  $\tau_{,2} = \tau_{,4} = 0$ . The corresponding retarded time has the form [18]  $\tau = t - r - ia\cos\theta$ , which allows us to obtain the general retarded-time solution  $\gamma = \gamma_0 + \gamma_f$  as the sum of the partial solution  $\gamma_0$  containing series of poles,  $\gamma_0 = \sum_i c_i(\tau) \frac{1}{P^2 Y(Y - Y_i)}$  with oscillating factors  $c_i(\tau) = i\omega^{21/2} q_i(\tau) \exp\{i\omega_i\tau\}$ , and the term  $\gamma_f = \frac{\phi(Y, \tau)}{P}$  which is determined by a free function  $\phi(Y, \tau)$ .

The free term  $\gamma_f$  could be taken with the same series of poles and the same oscillating factors of opposite sign to provide regularization of poles by subtraction. However, the extra slowly varying function  $P = 2^{-1/2}(1 + \bar{Y}Y)$  has the different degrees in the front of the functions  $\phi$  and  $\psi$ , which prevents immediate compensation. Working in a complex vicinity of  $i$ th pole, one can set  $\bar{Y} = \bar{Y}_i$  and expand the function  $P(\bar{Y}_i, Y)$ , in  $(Y - Y_i)$ ,

$$P(\bar{Y}_i, Y) = P_i + 2^{-1/2}\bar{Y}_i(Y - Y_i) + \mathcal{O}[(Y - Y_i)^2], \quad (\text{B.5})$$

where  $P_i = 2^{-1/2}[1 + \bar{Y}_i Y_i]$ . Then the free function  $\phi(Y, \tau)$  chosen in the form  $\phi(Y, \tau) = -\sum_i c_i(\tau) \frac{1}{Y_i P(\bar{Y}_i, Y)(Y - Y_i)}$ , will regularize the solution, compensating the poles in  $\gamma_0$ . The result of this compensation is the function

$$\gamma_{\text{reg}} = -\sum_i c_i(\tau) \frac{2^{-1/2}\bar{Y}_i + \mathcal{O}(Y - Y_i)}{Y_i P P(\bar{Y}_i, Y)}. \quad (\text{B.6})$$

Such a regularization does not touch the function  $A$  and provides a uniform low-frequency limit  $\gamma_{\text{out}} \rightarrow 0$ . The regularized function  $\gamma_{\text{out}}$  represents a stochastic process which is the sum of many oscillating terms. The mean value of this process  $\langle \gamma \rangle$  is zero. Looking on the product  $A\bar{\gamma}$ , one sees that the  $i$ th pole in function  $A$  is compensated by the  $k$ th zero  $(Y - Y_k)$  in  $\bar{\gamma}$  by setting  $i = k$ , while for  $i \neq k$ , the product averaged over time  $A\bar{\gamma}$  vanishes due to the difference in the frequencies of oscillations. Therefore, we pick up the exact wave solutions of the electromagnetic sector, for which the correlator  $\langle A\bar{\gamma} \rangle$  contains a sum of regular terms tending to zero in the low-frequency limit. Similar treatment of the correlator  $\langle \gamma_{\text{out}} \gamma_{\text{out}} \rangle$  shows that the terms with  $i \neq k$  do not correlate and vanish, while the sum with  $i = k$  survives, displaying radiation along the Kerr congruence, which is similar to the radiation of the Vaidya shining star solution.

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<sup>6</sup> These condition means that the gradient  $d\tau$  is spanned by the null vectors  $e^1$  and  $e^3$ , and that the complex planes  $\tau = \text{const}$  are null and tangent to the light cones. Note also, that these constraints extend to all the projective twistor coordinates  $Z^A$  [33].

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