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Alcock-Paczyński Test with Model-independent BAO Data

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Cosmological tests based on the statistical analysis of galaxy distributions usually depend on source evolution. An exception is the Alcock-Paczyński (AP) test, which is based on the changing ratio of angular to spatial/redshift size of (presumed) spherically-symmetric source distributions with distance. Intrinsic redshift distortions due to gravitational effects may also have an influence, but they can now be overcome with the inclusion of a sharp feature, such as the Baryonic Acoustic Oscillation (BAO) peak. Redshift distortions affect the amplitude of the peak, but impact its position only negligibly. As we shall show here, the use of this diagnostic, with new BAO peaks from SDSS-III/BOSS at average redshifts $\langle z\rangle = 0.38$, 0.61 and 2.34, disfavors the current concordance (ΛCDM) model at 2.3σ. Within the context of expanding Friedmann-Robertson-Walker (FRW) cosmologies, these data instead favor the zero active mass equation-of-state, $\rho + 3p = 0$, where ρ and p are, respectively, the total density and pressure of the cosmic fluid, the basis for the $R_{\rm h} = ct$ universe.

Keywords: cosmology: cosmological parameters – cosmology: distance scale – cosmology: observations – quasars: general

1. Introduction

A highly desirable goal in cosmology is the acquisition of model-independent data that can be used to test theoretical models and to optimize their parameters. There are two main types of cosmological observations that shed light on the geometry of the Universe: a measurement of the fluctuations in the Cosmic Microwave Background Radiation (CMB) and the analysis of large-scale structure via the inferred distribution of galaxies. The quality and quantity of both kinds of data have progressed significantly in recent decades. CMB anisotropies provide the most evident support for the concordance (ΛCDM) model, but one should find an independent confirmation of this theory and its parameters because CMB anisotropies may be generated/modified by mechanisms other than those in the standard picture[1,2,3,4], and may also be contaminated by other effects[5]. Cosmological tests using surveys of galaxies have also been developed to provide information on the geometry of the Universe: for instance, the Hubble diagram or the angular-size test. However, both of these need some assumptions in order to provide cosmological information.

As of today, Hubble diagrams constructed from the apparent magnitude (taking into account K-corrections) vs. distance or redshift of first-rank elliptical galaxies in clusters require a strong evolution of galaxy luminosities to fit FRW cosmologies[6]. For Type Ia Supernovae[7] (SNIa) embedded in these galaxies, or for gamma-ray bursts[8], one assumes zero evolution, and the standard model fits the data, though some systematic effects may be affecting the result, including a possible evolution in the metallicity of SNIa progenitors[9], possible internal extinctions[10], time variations of the grey-dust absorption of light from these supernovae in various types of host galaxies[11], and observational selection effects. The angular-size vs. redshift test has been developed by several authors[12], using sources seen at radio, near-infrared and visible wavelengths. All these applications produce an angular size $\theta \sim z^{-1}$ up to $z \sim 3$, suggesting a strong evolution in galactic radii. The fact that galaxies with the same luminosity apparently were six times smaller at $z = 3.2$ than at $z = 0$ makes it difficult to compare different models.[13] The surface brightness (known as the 'Tolman') test also depends strongly on the assumption of galaxy evolution, so the results of this test[14,15] may vary hugely depending on one's interpretation.

An alternative cosmological test based on the Alcock & Paczyński (AP)[16,17] approach evaluates the ratio of observed angular size to radial/redshift size. This probe is based on the assumption that the distribution of sources is spherical, regardless of distance, and large, so that their size can be easily measured over a broad range of redshifts, preferably extending beyond $2-3$. Appropriate targets therefore include clusters of galaxies, rather than the individual galaxies themselves, like in some of the other tests described above. The main advantage of the AP test is that, regardless of whether or not galaxies may have evolved with redshift, it depends only on the geometry of the Universe. However, redshift distortions induced by peculiar velocities[18,19,20], described in linear perturbation theory by the parameter β, can also have an influence. In the linear regime, the value of β measured from redshift distortions corresponds to the solution of the linearized continuity equation ($\beta\delta + \nabla\vec{v} = 0$, where δ is the relative overdensity, and $\vec{v}$ is the velocity field)[20].

There is now a way to overcome this possible contamination—via the inclusion in the AP test of an effect with a sharp feature, such as the Baryon Acoustic Oscillation (BAO) peak, for which the degeneracy between redshift and geometric and gravitational distortions is almost completely broken[21]. The reason is that the value of β affects primarily the amplitude of the peak, but its impact on the position is negligible. In this paper, we carry out the AP test using three Baryon Acoustic Oscillation (BAO) peak positions that significantly increase its precision over what has been achieved thus far. In so doing, we demonstrate that the concordance (ΛCDM) model is disfavored by these new data. Instead, the AP test favors a model with zero active mass, i.e., with the equation of state $\rho + 3p = 0$, where ρ and p are, respectively, the total density and pressure of the cosmic fluid[22,23,24,25].

2. The Alcock-Paczyński Test

Given a spherically symmetric structure, or distribution of objects, with radius

$$s_{\parallel} = \Delta z \frac{d}{dz} d_{\rm com}(z) \tag{1}$$

along the line of sight and a radius

$$s_{\perp} = \Delta\theta (1+z)^m d_{\rm A}(z) \tag{2}$$

(where $m = 1$ with expansion, while $m = 0$ for a static universe) perpendicular to the line of sight, the ratio

$$y \equiv \frac{\Delta z}{z\Delta\theta} \frac{s_{\perp}}{s_{\parallel}} \tag{3}$$

depends only on the cosmological comoving distance, $d_{\rm com}(z)$, and the angular-diameter distance, $d_{\rm A}(z)$, and is independent of any source evolution.

Previous applications of the galaxy two-point correlation function to measure a redshift-dependent scale for the determination of $y(z)$ were limited by the difficulty in disentangling the acoustic length in redshift space from redshift distortions due to internal gravitational effects[17]. A serious drawback with this process is that inevitably one had to either pre-assume a particular model, or adopt prior parameter values, in order to estimate the level of contamination. And the wide range of possible distortions (i.e., values of β; see, e.g., Eqns. 4 and 5 in ref. [17]) for the same correlation-function shape resulted in seriously large errors.

But this situation has changed dramatically in the past few years with the use of reconstruction techniques[26,27] that enhance the quality of the galaxy two-point correlation function and the much more precise determination of the Ly-α and quasar auto- and cross-correlation functions, resulting in the measurement of BAO peak positions to better than $\sim 4\%$ accuracy. In this paper, we determine $y(z)$ using three BAO peak positions: 1) the measurement of the BAO peak position in the anisotropic distribution of SDSS-III/BOSS DR12 galaxies[28] at the two independent/non-overlapping bins with $\langle z \rangle = 0.38$ and $\langle z \rangle = 0.61$, in which a technique of reconstruction to improve the signal/noise ratio was applied. This technique affects the position of the BAO peak only negligibly, so the measured parameters are independent of any cosmological model. And 2) the self-correlation of the BAO peak in the Ly-α forest in the SDSS-III/BOSS DR11 data[29] at $\langle z \rangle = 2.34$, plus the cross-correlation of the BAO peak of QSOs and the Ly-α forest in the same survey[21].

In all of these measurements, the template used for the correlation function was drawn from the concordance model. However, the actual shape of the BAO peak does not significantly affect the calculation of its centroid position, both along the line-of-sight and in the direction perpendicular to it, when its FWHM is very narrow. Any shape could be used, and the results would be the same. The peak's narrowness strongly reduces the impact of redshift distortions, which affect the peak's amplitude, but not so much its location. Any modifications to this amplitude as a function of distance may produce a second-order shift to the centroid, but always negligibly. Reconstruction also significantly reduces the effects of redshift-space distortions at the BAO scale, isotropizing the correlation function[27]. This conclusion is reflected, for instance, in the fact that, although the errors for the parameter β quoted in Table 2 of ref. [29] are very large, the relative error bars for $d_A(z)$ and $H(z)$ are much smaller. If the BAO peak measurements were sensitive to β, their error bars would be much bigger. The redshift distortions produce systematic errors of order of 1-2% in $d_A(z)$ and $H(z)$[30], which are already included in the errors we use for the data, though some authors[31] give a slightly more pessimistic estimation of the error associated with the Hubble parameter. But this situation has improved even more recently[32], with the report of systematic uncertainties in D_A/H that are even smaller: 0.5% for the AP parameter and 0.4% for the distance scale—and these were considered conservative. This impressive precision amplifies the probative power of these measurements, given that statistical errors as small as $\sim 4\%$ are now achievable.

The angular-diameter distance $d_A(z)$ and Hubble constant $H(z)$ are related to the variable $y(z)$ via the expression

$$y(z) = \left(1 + \frac{1}{z}\right) \frac{d_A(z)^* H(z)}{c} \,, \tag{4}$$

where $d_A(z)^*$ is the measured angular-diameter distance assuming an expanding Universe, that is, $d_A(z)^* = d_A(z)$ if there is expansion and $d_A(z)^* = \frac{d_A(z)}{(1+z)}$ for a static universe. Very importantly, the quantity $y(z)$ is independent of the uncertain (co-moving) acoustic scale r_s, since $d_A(z)^* \propto r_s$, while $H(z) \propto 1/r_s$, so the dependence on r_s cancels out in the product. (Sometimes an alternative definition of this ratio, $F_{AP}(z) \equiv zy(z)$, has been used in the literature, but these properties are the same for both working definitions.)

At low redshift, the values of the $F_{AP}(z)$ of ref. [28] lead directly to $y(z = 0.38) = 1.079 \pm 0.042$, $y(z = 0.61) = 1.248 \pm 0.044$. At high redshift, we have $c/H(z = 2.34)r_s = 9.15^{+0.20}_{-0.21}$ and $d_A(z = 2.34)/r_s = 10.93^{+0.35}_{-0.34}$ (see Eqns. 22 and 23 in ref. [29]), plus the adoption of a correlation coefficient[29] of -0.6 between c/H and d_A, which lead to $y(z = 2.34) = 1.706 \pm 0.083$. The uncertainties in $y(z)$ are found by propagating the errors in $H(z)$ and $d_A(z)$ through Equation (4) including the covariance terms derived from the given correlation coefficients.

These three are the only BAO measurements currently available that measure both $d_A(z)$ and $H(z)$ with small statistical errors. Other reported values (e.g., ref. [33]) are older, less accurate versions of these same BOSS measurements. The available WiggleZ data[34] are not included because they are much less accurate than BOSS data: they have relative error bars of 20-30% for $y(z)$. And we do not include clustering measurements of $d_A(z)$ and $H(z)$ based on the anisotropic two–point correlation function at shorter scales[35,36,37,38,39,40] because these are biased by the pre-assumption of a cosmological model, or the adoption of priors, used to characterize the internal redshift distortions (i.e., the parameter β).

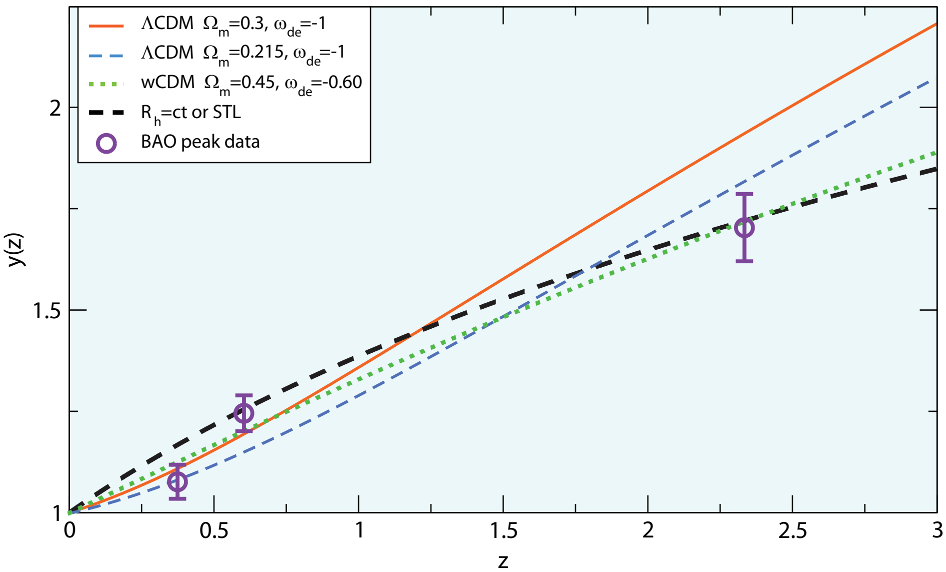

Fig. 1. Plot of $y(z)$ (the ratio of observed angular size to radial/redshift size) versus z, inferred from the data using the BAO peak measurements. This figure also shows the predictions of the concordance (ΛCDM) model with fiducial parameter values $\Omega_{\rm m} = 0.3$ and $w_{\rm de} \equiv w_\Lambda = -1$, a best fit using $\Omega_{\rm m}$ as a free parameter (but with a fixed $w_\Lambda = -1$), and a best fit when both $\Omega_{\rm m}$ and $w_{\rm de}$ are free. The (heavy) dashed curve represents the (expanding) $R_{\rm h} = ct$ and (static) STL cosmologies, whose $y(z)$ functions are (coincidentally) identical (see text).

In figure 1, we compare various model predictions with these three BAO measurements of $y(z)$, following the conventional methodology of the AP cosmological test[16,17]. We here plot the function for the wCDM model (the version of ΛCDM with a dark-energy equation of state $w_{\rm de} \equiv p_{\rm de}/\rho_{\rm de}$ different from $w_\Lambda = -1$), in addition to the predictions of the standard ΛCDM model itself with fiducial parameter values $\Omega_{\rm m} = 0.3$ and $w_{\rm de} \equiv w_\Lambda = -1$. Additional comparisons are provided using ΛCDM with a non-concordance value of $\Omega_{\rm m}$ as the sole free parameter, and also using fits with both $\Omega_{\rm m}$ and $w_{\rm de}$ free. Table 1 summarizes the quality of the fits plotted in figure 1. Additionally, we plot $y(z)$ for the $R_{\rm h} = ct$ cosmology[22,23,24,25], in which

$$d_{\rm com}(z) = (1+z)d_A(z) = \frac{c}{H_0}\ln(1+z)\;, \tag{5}$$

and has a $y(z)$ coincident with that of the Static/Tired Light (STL) model[17], though its representation of the Universe is based on the FRW metric with expansion, while STL is static. This coincidence arises because for STL $d_{\rm com}(z) = d_A(z) = \frac{c}{H_0}\ln(1+z)$, which corresponds to a factor $(1+z)$ difference with the angular diameter distance $d_A(z)$ in the $R_{\rm h} = ct$ universe. The latter also has a Hubble parameter $H(z) = H_0(1+z)$, while H is constant in STL. So the various factors of $(1+z)$ all cancel in the formulation of $y(z)$ when calculated with $d_A^*(z)$ according to Equation (4). The $R_\mathrm{h} = ct$ model represents a cosmology with zero 'active mass,' i.e., with the equation of state $\rho + 3p = 0$, where ρ and p are, respectively, the total density and pressure of the cosmic fluid. This model has successfully passed all other cosmological tests applied to it thus far[41,42,43,44,45,46,47,48], though there remain some observations to be understood.

Table 1. Results of the χ^2-test, using the $N = 3$ points of the BAO peak data: the minimum reduced $\chi^2_\mathrm{red} \equiv \chi^2/(N-\nu)$, where ν is the number of free parameters; best-fit free parameters (if any); and associated probability of the models.

Model	$\chi^2_\mathrm{red,min}$	Free parameters	Probability
ΛCDM; $\Omega_\mathrm{m} = 0.3$, $w_\mathrm{de} = -1$	3.25	—	0.0207
ΛCDM; Ω_m free; $w_\mathrm{de} = -1$	3.31	$\Omega_\mathrm{m} = 0.215^{+0.045}_{-0.040}$	0.0367
wCDM; Ω_m, w_de free	2.38	$\Omega_\mathrm{m} = 0.45^{+0.21}_{-0.19}$, $w_\mathrm{de} = -0.60^{+0.30}_{-0.27}$	0.127
$R_\mathrm{h} = ct$ (or STL)	1.58	—	0.192

3. Discussion

3.1. *Model Comparison with the Alcock-Paczyński Test*

In Table 1, we list the outcome of our model comparisons based on the optimized fits to the data shown in figure 1, using a minimization of the χ^2 statistic and a calculation of the probability of each model being correct when the relative number of free parameters is taken into account. Each is an absolute probability, independent of the other models, and the confidence limits are extracted from the χ^2-distribution. The $R_\mathrm{h} = ct$ universe (and, coincidentally, also the static STL cosmology) fits the data very well without any ad hoc optimization of free parameters. For the ΛCDM/wCDM cosmology, we see that the fiducial parameter values ($\Omega_\mathrm{m} = 0.3$, $w_\Lambda = -1$) are excluded at a 97.93% C.L. (equivalent to 2.3σ), while even allowing Ω_m to be free with a fixed $w_\Lambda = -1$ (best fit for $\Omega_\mathrm{m} = 0.215^{+0.045}_{-0.040}$) is excluded at 1.7σ. The only way to reconcile the data with the wCDM cosmology at an exclusion C.L. $< 95\%$ is to set $w_\mathrm{de} = -0.60^{+0.30}_{-0.27}$, higher than the fiducial value $w_\mathrm{de} = -1$, and $\Omega_\mathrm{m} = 0.45^{+0.21}_{-0.19}$.

The confidence level contours for the standard model are shown in figure 2. Note that the probabilities given in Table 1 are $P(\chi^2, N-\nu)$. However, in figure 2 the probabilities of the fits are calculated as $P(\chi^2, N)$ since the parameters are here assumed to have prior values. That is, in figure 2 we are not taking into account the

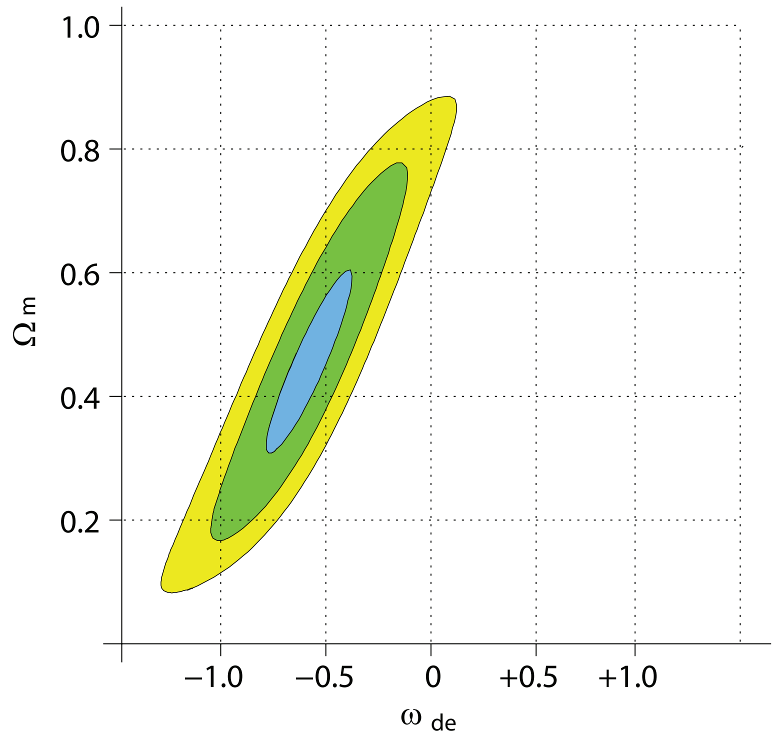

Fig. 2. Confidence level contours as a function of Ω_m and $w_{\rm de}$ for the ΛCDM/wCDM model. The three levels are equivalent to 1-3 σ in a Gaussian distribution (i.e., 68.3% C.L., cyan; 95.4% C.L., green; 99.73% C.L., yellow). The concordance (ΛCDM) model corresponds to the optimized parameter values $\Omega_m = 0.3$ and $w_{\rm de} = -1$, which are excluded at the 97.93% C.L.

reduction of the probability due to a reduction in the number of degrees of freedom, since the parameters are not being optimized freely. It is done this way in order to compare the outcome using standard values ($\Omega_{\rm m} = 0.3$, $w_{\rm de} = -1$) with those where the parameters are free.

Our results reinforce the conclusions drawn in refs. [29,49,50], who also remarked on the tension between the measurement at $z = 2.34$ and predictions of the standard model, based on various methods of analysis. Abdalla et al.[49] also pointed out that values of $w_{\rm de} > -1$ were necessary to alleviate the tension for the datum $H(z = 2.34)$, while Aubourg et al.[50] realized this tension, but did not explore such possible solutions because they fitted the cosmological parameters using a combination of BAO+(Supernovae data and/or the CMB). However, our results are based on the simultaneous analysis of the available model-independent BAO data (at $\langle z\rangle = 0.38$, $z = 0.61$ and 2.34), which more emphatically exclude the standard model, while favoring the $R_{\rm h} = ct$ universe. In this paper, we do not combine the AP test data with the CMB or Supernova data, because this step would dilute the information concerning the anomaly reflected in our results.

Our conclusions are stronger than the tension reported in refs. [29,49,50], for the principal reason that the Alcock-Paczyński test amplifies the deviation of the measurements relative to the ΛCDM prediction. These authors reported a tension at the $\gtrsim 2\,\sigma$ level. The AP test, however, compares the observed product $d_A(z)\,H(z)$ with that of the model, and since both BAO measurements of $d_A(z)$ and $H(z)$ are higher than those expected in the standard model, the AP discrepancy is larger than simply that of the quadrature sum of the two individual discrepancies. And while Aubourg et al.[50] did not find any models that substantially improved the agreement between theory and the Ly-α forest BAO measurement without worsening the corresponding fit to other data, they did not consider the $R_{\rm h}=ct$ cosmology. Our results here show that, insofar as the AP test by itself is concerned, this model (and coincidentally the STL model) does in fact yield a better fit to the BOSS measurements than does the standard model.

A comparison between these results and those reported in ref. [17] highlights the dramatic improvement in the measurement of the BAO peak position that has led to the important conclusions drawn in this paper. Though the measurements of $y(z)$ based on the galaxy two-point correlation function used in that earlier work were good enough to rule out all but the concordance and STL models (the $R_{\rm h}=ct$ universe was not included in that comparison, but its $y(z)$ function is identical to that of STL), the errors arising from the contamination due to internal redshift distortions were still too large for us to discriminate between these two cosmologies. As we can see from figure 1, however, the significant improvement in the precision with which the BAO peak position is measured now[28,29] disfavors the standard model.

3.2. *The Acoustic Scale*

The surprising and emphatic result summarized in figure 1 now compels us to examine one of ΛCDM's most impressive successes—the interpretation of the CMB and BAO lengths as arising from a single, consistent acoustic scale—and whether this identification survives into the $R_{\rm h}=ct$ framework. An acoustic angular size, $\theta_s=(0.596724\pm0.00038)^\circ$, has been measured in both the temperature and polarization power spectrum, most recently with *Planck*[51]. This 'standard ruler' is believed to be responsible not only for the multi-peak structure in the CMB power spectrum, but also for leaving its indelible imprint on the large-scale structure we see today. In many ways, the optimization of the parameters in ΛCDM relies critically on the correct interpretation of this 'sound horizon.'

One may question some of the assumptions made in calculating the acoustic scale in ΛCDM, including its estimation from the comoving distance r_s traveled by a sound wave rather than from an actual solution to the geodesic equation in an expanding medium, but the fact that $r_s^{\rm CMB}/r_s^{\rm BAO}\sim 1$ is evidence in support of the standard model. Let us now see what happens in the $R_{\rm h}=ct$ universe. The measured value of $r_s^{\rm BAO}$ at $z=2.34$ is $\sim 144.8\pm8$ Mpc, assuming the *Planck* Hubble constant $H_0 = 67.8$ km s^{-1} Mpc^{-1} (ref. [51]). We have recently initiated an analysis of the CMB power spectrum and its angular correlation function[52] in the context of $R_\mathrm{h} = ct$, which has yielded an optimized value of $\approx 3.2/2\pi$ for the ratio $u_\mathrm{min}/2\pi$ of the angular-diameter distance $d_A(t_\mathrm{dec})$ over the maximum fluctuation size λ_max at decoupling. In $R_\mathrm{h} = ct$, $\lambda_\mathrm{max} = 2\pi c/H(t_\mathrm{dec})$, where $H(t_\mathrm{dec})$ is the Hubble constant at the last scattering surface. Therefore, the comoving CMB acoustic scale in this model is given by the simple expression $r_s^\mathrm{CMB} = \theta_s(c/H_0)u_\mathrm{min}$ (remembering that $H(t) = 1/t$ and $(1+z) = t_0/t$ in this cosmology). That is, $r_s^\mathrm{CMB} \approx 148$ Mpc, which is fully consistent with the BAO value 144.8 ± 8 Mpc measured at $z = 2.34$.

If the results of the AP test are reliable, and $R_\mathrm{h} = ct$ is indeed the correct cosmology, it is therefore likely that the CMB and BAO length scales have a common origin, just as they apparently do in ΛCDM as well. It is therefore highly desirable to carry out additional high-precision measurements of the BAO scale using a broader redshift coverage than is currently available. A quick inspection of figure 1 suggests that to distinguish between the various models, the AP test is best suited to redshifts $z \gtrsim 2$—the higher the better.

4. Conclusions

The results of our analysis in this paper show that, if the measurements of $d_A(z)$ and $H(z)$ derived from the BAO peak anisotropic distributions are correct, the concordance ΛCDM model, optimized to fit SNIa and CMB data, does not pass the AP test. In light of the AP results, one may begin to question its status as a true 'concordance' model. Instead, the AP test using these model-independent data favors the $R_\mathrm{h} = ct$ universe, which has thus far also been favored by model selection tools in other one-on-one comparative tests with ΛCDM[41,42,43,44,45,46,47,48,53,54]. The consequences of this important result are being explored elsewhere, including the growing possibility that inflation may have been unnecessary to resolve any perceived 'horizon problem' and therefore may have simply never happened[43].

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References

1. J. V. Narlikar, G. Burbidge and R. G. Vishwakarma, *JApA* **28** (2007) 67.
2. G. W. Angus and A. Diaferio, *Mon. Not. R. Astron. Soc.* **417** (2011) 941.
3. M. López-Corredoira, *Int. J. Mod. Phys. D* **22** (2013) id. 1350032.
4. F. Melia, *Astron. Astrophys.* **561** (2014) id. A80.
5. M. López-Corredoira, *J. Astrophys. Astron.* **28** (2007) 101.
6. D. Schade, L. F. Barrientos and O. López-Cruz, *Astrophys. J. Lett.* **477** (1997) L17.
7. M. Kowalski *et al.*, *Astrophys. J.* **686** (2008) 749.
8. H. Wei, *J. Cosmol. Astropart. Phys.* **8** (2010) 20.
9. P. Podsiadlowski *et al.*, *New Astron. Rev.* **52** (2008) 381.
10. L. G. Balázs *et al.*, *Astron. Nachrichten* **327** (2006) 917.
11. A. I. Bogomazov and A. V. Tutukov, *Astron. Reports* **55** (2011) 497.
12. K. P. Kapahi, in *Observational Cosmology* (IAU Symp. 124), eds. A. Hewitt, G. Burbidge and L. Z. Fang (Reidel, Dordrecht, 1987), p. 251.
13. M. López-Corredoira, *Int. J. Mod. Phys. D* **19** (2010) 245.
14. L. M. Lubin and A. Sandage, *Astron. J.* **122** (2001) 1084.
15. E. J. Lerner, R. Falomo and R. Scarpa, *Int. J. Mod. Phys. D* **23** (2014) id. 1450058.
16. C. Alcock and B. Paczyński, *Nature* **281** (1979) 358.
17. M. López-Corredoira, *Astrophys. J* **781** (2014) 96.
18. N. Kaiser, *MNRAS* **227** (1987) 1.
19. T. Matsubara and Y. Suto, *ApJL* **470** (1996) L1.
20. A.J.S. Hamilton AJS, The Evolving Universe. Selected Topics on Large-Scale Structure and on the Properties of Galaxies, ed. D. Hamilton, Kluwer, Dordrecht (1998) 185.
21. A. Font-Ribera et al., *JCAP* **5** (2014) id27.
22. F. Melia, *Mon. Not. R. Astron. Soc.* **382** (2007) 1917.
23. F. Melia, *Front. Phys.* **11(4)** (2016) Id. 119801 (arXiv:1601.04991).
24. F. Melia, *Front. Phys.* in press (2016) arXiv:1602.01435.
25. F. Melia and A. S. H. Shevchuk, *Mon. Not. R. Astron. Soc.* **419** (2012) 2579.
26. D. J. Eisenstein, H.-J. Seo, E. Sirko and D. N. Spergel, *ApJ* **664** (2007) 675.
27. N. Padmanabhan et al., *MNRAS* **427** (2012) 2132.
28. S. Alam et al., *MNRAS* submitted (2016) [arXiv.org:1607.03155]
29. T. Delubac et al., *Astron. Astrophys.* (2015) 59.
30. A. Taruya, T. Nishimichi and S. Saito, *Phys. Rev. D* **82** (2010) 063522.
31. C. G. Sabiu and Y.-S. Song, *MNRAS* in press (2016) arXiv:1603.02389.
32. S. Alam, M. Ata, S. Bailey, F. Beutler, D. Bizyaev et al., *MNRAS* submitted (2016) arXiv:1607.03155
33. L. Anderson L et al., *MNRAS* **441** (2014) 24.
34. S. Hinton (2016) arXiv:1604.01830.
35. C.-H. Chuang and Y. Wang, *MNRAS* **426** (2012) 226.
36. C. Blake et al., *MNRAS* **425** (2012) 405.
37. B. A. Reid et al., *MNRAS* **426** (2012) 2719.
38. A. G. Sánchez et al., *MNRAS* **440** (2014) 2692.
39. L. Samushia et al., *MNRAS* **439** (2014) 3504.
40. F. Beutler et al., *MNRAS* **443** (2014) 1065.
41. F. Melia, *Astrophys. J.* **764** (2013) 72.
42. J.-J. Wei, X. Wu and F. Melia, *Astrophys. J.* **772** (2013) 43.
43. F. Melia, *Astron. Astrophys.* **553** (2013) A76.
44. F. Melia, *Astron. J.* **147** (2014) 120.
45. J. J. Wei, X. Wu and F. Melia, *Astrophys. J.* **788** (2014) 190.

46. F. Melia, J.-J. Wei and X. Wu, *Astron. J.* **149** (2015) 2.
47. F. Melia, *Astron. J.* **149** (2015) 6.
48. J. J. Wei, X. Wu and F. Melia, *Mon. Not. R. Astron. Soc.* **447** (2015) 479.
49. E. Abdalla, E.G.M. Ferreira, J. Quintin and B. Wang, (2014) arXiv:1412.2777.
50. E. Aubourg et al., (2014) arXiv:1411.1074.
51. Planck Collaboration, P.A.R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi et al., *A&A* **571** (2014) id.A23.
52. F. Melia, R. T. Gènova-Santos and M. López-Corredoira, 2016, MNRAS, in preparation.
53. F. Melia and R. S. Maier, *MNRAS* **432** (2013) 2669.
54. J.-J. Wei, X. Wu and F. Melia F., *MNRAS* **439** (2014) 3329.