

Hubble tensions: a historical statistical analysis

Martín López-Corredoira^{1,2}★

¹ *Instituto de Astrofísica de Canarias, E-38205 La Laguna, Tenerife, Spain*

² *Departamento de Astrofísica, Universidad de La Laguna, E-38206 La Laguna, Tenerife, Spain*

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ABSTRACT

Statistical analyses of the measurements of the Hubble-Lemaître constant H_0 (163 measurements between 1976 and 2019) show that the statistical error bars associated with the observed parameter measurements have been underestimated—or the systematic errors were not properly taken into account—in at least 15–20% of the measurements. The fact that the underestimation of error bars for H_0 is so common might explain the apparent discrepancy of values, which is formally known today as the Hubble tension. Here we have carried out a recalibration of the probabilities with this sample of measurements. We find that $x\sigma$ deviation is indeed equivalent in a normal distribution to $x_{\text{eq.}}\sigma$ s deviation in the frequency of values, where $x_{\text{eq.}} = 0.83x^{0.62}$. Hence, a tension of 4.4σ , estimated between the local Cepheid–supernova distance ladder and cosmic microwave background (CMB) data, is indeed a 2.1σ tension in equivalent terms of a normal distribution of frequencies, with an associated probability $P(> x_{\text{eq.}}) = 0.036$ (1 in 28). This can be increased up to a equivalent tension of 2.5σ in the worst of the cases of claimed 6σ tension, which may anyway happen as a random statistical fluctuation.

Key words: cosmological parameters — Cosmology: observations — distance scale

1 INTRODUCTION

The Hubble–Lemaître constant, H_0 , is one of the fundamental cosmological parameters. We know its value approximately, but there is not yet a consensus regarding an accurate estimation of the parameter.

From the beginning of the discovery of the apparent magnitude relation of the galaxies in the 1920s, first by Lemaître and later by Hubble (the so-called Hubble–Lemaître diagram), H_0 has decreased the value of that constant by almost an order of magnitude. In the 1980s, two preferred values were defended by different teams: either 50 or $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Later, in the 1990s and 2000s, a value of around $70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ became dominant, with preference for the value of $72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ obtained by the Hubble Space Telescope (HST) Key Project using supernovae (Freedman & Madore 2010). Nonetheless, discordant values were later published. A period–luminosity bias for the calibration of distances with nearby galaxies would justify a Hubble–Lemaître constant to be reduced to values of around $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Paturel 2008; Sandage, Tammann & Reindl 2009). Even supernova data with HST were fitted with these values. As to the possible (non-)universality of the Cepheid period–luminosity relation, it was argued that low metallicity Cepheids have flatter slopes, and that the derived distance would depend on what relation is used (Tammann, Sandage & Reindl 2003).

We must also bear in mind that the value of H_0 is determined without knowing on which scales the radial motion of galaxies and clusters of galaxies relative to us is completely dominated by the Hubble–Lemaître flow (Matravers, Ellis & Stoeger 1995). The homogeneity scale may be much larger than expected (Yadav, Bagla & Khandai 2010; Sylos Labini 2011), thus giving important net velocity flows on

large scales that are incorrectly attributed to cosmological redshifts. Also, values of H_0 derived from cosmic microwave background radiation (CMBR) analyses are subject to the errors in the cosmological interpretation of this radiation (López-Corredoira 2013). That is, CMBR data are interpreted under the assumption of some model, which relates Λ CDM cosmological model with the power spectrum of CMBR, and there is the possibility that CMBR can be interpreted with a different model. For instance, a Friedmann-Lemaître-Robertson-Walker metric lacking dark energy would also produce a big change in the values of H_0 (Blanchard et al. 2003). Moreover, Galactic foregrounds are not perfectly removed (López-Corredoira 2007; Axelsson et al. 2015; Creswell & Naselsky 2021), and these are another source of uncertainties.

Yet another controversy arose more recently on the value of H_0 . The Hubble–Lemaître constant estimated from the local Cepheid–supernova distance ladder is at odds with the value extrapolated from CMB data, assuming the standard cosmological model, 74.0 ± 1.4 (Riess et al. 2019) and $67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Planck Collaboration 2020) respectively, which gives an incompatibility at the 4.4σ level. This tension can even be increased up to 6σ depending on the datasets considered (Di Valentino, Mena & Pan 2021). Given the number of systematic errors that may arise in the measurements, this should not be surprising. However, this problem has motivated hundreds of papers since 2019 and many solutions to the problem have been proposed—see the reviews in Di Valentino, Mena & Pan (2021); Perivolaropoulos & Skara (2021); Abdalla et al. (2022)—discussing either the method to estimate H_0 or new theoretical scenarios.

Here we will not contribute with a new solution to this Hubble tension in physical terms. Instead we carry out an historical investigation

★ E-mail: martin@lopez-corredoira.com

to determine whether or not the given error bars truly represented the dispersion of values in a historical compilation of H_0 values. We also show how we can use this knowledge to recalibrate the probabilities of some tension of this kind.

2 BIBLIOGRAPHICAL DATA AND STATISTICAL ANALYSIS

We use the compilation of 163 values for H_0 between the years of 1976 and 2019 by [Faerber & López-Corredoira \(2020\)](#), where 120 measurements between 1990-2010 were taken from the previous compilation by [Croft & Dailey \(2015\)](#),¹ plus other 8 measurements between 1976 and 1989 and 35 measurements between 2011 and 2019. [Croft & Dailey \(2015\)](#) have made use of the NASA Astrophysics Data System to generate the dataset by carrying out an automated search of publication abstracts for the years 1990-2010, limiting the search to published papers which include values of H_0 and their error bars in the paper abstract itself. According to the authors, although this selection does not cover the 100% of the whole literature, it gathers most the measurements; this is representative and should not produce any statistical bias. The extra 43 measurements in [Faerber & López-Corredoira \(2020\)](#) obey a similar selection method.

The latest measurement we use is the value given by [Riess et al. \(2019\)](#), which marked the origin of the present-day Hubble tension. Our goal is investigating the historical records previous to 2019, and not the most recent ones, because we want to investigate how common the kind of tension pointed out by [Riess et al. \(2019\)](#) was until then. We wonder whether a 4.4σ is something never previously seen or something ordinary. The research of data after [Riess et al. \(2019\)](#)'s paper in 2019 is not the scope of this paper, as this would belong to an analysis of the post-Hubble tension epoch.

The correlation factor of H_0 with time² is $c = 0.027 \pm 0.013$, a 2σ significant correlation ([Faerber & López-Corredoira 2020](#)). Two sigma correlation is not a high significant detection, it is possibly a statistical fluctuation. If a more significant correlation had been obtained, it would have been a proof that the measurements of the parameter are not independent and there are systematic common errors variable with time or confirmation biases.³

¹ All of the data except the measurement of $93 \pm 1 \text{ km s}^{-1} \text{ Mpc}^{-1}$ by [Chiba & Yoshii \(1995\)](#), which we excluded for being $> 20\sigma$ away from the average.

² For two independent variables X and Y , the correlation factor is defined as $c = \frac{\langle XY \rangle}{\langle X \rangle \langle Y \rangle} - 1$, with error $Err(c) = \frac{\sigma_X \sigma_Y}{\sqrt{N} \langle X \rangle \langle Y \rangle}$. The Pearson correlation coefficient would be $\frac{c}{\sqrt{N} Err(c)}$.

³ In his famous lecture on 'Cargo Cult Science' (1974), Richard Feynman gave an example of Nobel Laureate Robert Millikan measuring the charge of an electron: "it's interesting to look at the history of measurements of the charge of the electron, after Millikan. If you plot them as a function of time, you find that one is a little bigger than Millikan's, and the next one is a little bit bigger than that, and the next ones a little bit bigger than that, until finally they settle down to a number which is higher" ([Feynman 1974](#)). Feynman goes on to ask why the final higher number was not discovered right away, and comes to the conclusion that when "[scientists] got a number that was too high above Millikan's, they thought something must be wrong and they would look for and find a reason why something might be wrong, leading them to eliminate values that were too far off, and did other things like that".

2.1 Analysis of 1976-2019 data

We neglect the variation with time of H_0 , and we continue our analysis with the weighted average. The weighted averages of the parameters in question were calculated by weighting each point by the inverse variance of that value:

$$\chi^2 = \sum_{i=1}^N \frac{(H_{0,i} - \overline{H_0})^2}{\sigma_i^2}. \quad (1)$$

We obtain $\overline{H_0} = 68.26 \pm 0.40 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\chi^2 = 575.7$ ([Faerber & López-Corredoira 2020](#)). For the value of χ^2 calculated using the weighted average, the probability that the observed trend is due to chance is $Q = 1.0 \times 10^{-47}$. In order to reach a value for Q that is statistically significant ($Q \geq 0.05$), 27 outliers (with more than 2.8σ deviation from the average value) must be removed from the data ($n = 136$, $\chi^2 = 161.3$), producing a value for Q of 0.061. These numbers are slightly different from those given by [Faerber & López-Corredoira \(2020\)](#), due to a minor error correction. Table 1 lists the 27 values with more than 2.8σ deviation. If instead of $68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ we took another value, the χ^2 would be larger, and the number of points to reject to get $Q \geq 0.05$ would also be larger.

Because of the simplifying assumption made about the neglect of the covariance of each observed measurement, the ratio of 27 in 163 is an approximation. A considerable fraction of the published measurements were not independent at all, but consisted of incremental updates to a chain of studies often based on a common foundation of measurements and assumptions. In general, treating the data as independent variables, as we do here, gives a conservative limit of the real dispersion of data, because the 'effective' number of degrees of freedom is lower than the number assuming independence, thus increasing the reduced χ^2 (i.e., decreasing even more the probability of this distribution of measurements). Nonetheless, there may be exceptions to this rule: for instance, if we take only two measurements with a large tension between them, and we multiply each of the points n times, the obtained Q with $2n$ points would be lower. However, this would happen only if the number of points that are multiplied are precisely those with a priori tension; if we multiply the points selected randomly, the value of Q obtained would be larger than the original value. Therefore, a very low Q might increase its value, but never within a range of $Q \geq 0.05$. That is, if we find a general tension in the data indicated by very low values of the probability Q , this tension would not be erased by considerations of covariance term analyses.

2.2 Analysis of the 2001-2019 data

The enormous discrepancies between cited uncertainties in derived values of H_0 and the much larger dispersions when values from different authors were compared to each other was already realized in the 1980s and 1990s (e.g., [Hanes 1982](#); [Rowan-Robison 1985](#); [Kennicutt, Freedman & Mould 1995](#)). We now know that there were several underlying reasons for this underestimation of uncertainties in H_0 in the measurements before the year ≈ 2000 : errors were usually estimated rather than measured directly via comparison of independent methods; and systematic errors were usually ignored and even the statistical errors were underestimated. Around the turn of the century, when beginning, for example, with the Hubble Space Telescope Key Project on the distance scale, authors began quantifying their uncertainties via multiple independent distance measurements, with separate explicit, and conservative, estimates of the remaining systematic errors (e.g., [Freedman et al. 2001](#)). However, looking at

Table 1. Outliers: measurements of H_0 in which $|H_0 - \overline{H_0}| > 2.8\sigma$, where $\overline{H_0} = 68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the weighted average of the 163 values of the literature.

Year	H_0 (km s ⁻¹ Mpc ⁻¹)	$\frac{ H_0 - \overline{H_0} }{\sigma}$	Authors
1976	50.3 ± 4.3	4.2	Sandage & Tammann
1984	45.0 ± 7.0	3.3	Jöeveer
1990	52.0 ± 2.0	8.1	Sandage & Tammann
1993	47.0 ± 5.0	4.3	Sandage & Tammann
1994	85.0 ± 5.0	3.3	Lu et al.
1996	84.0 ± 4.0	3.9	Ford et al.
1996	57.0 ± 4.0	2.8	Branch et al.
1996	56.0 ± 4.0	3.1	Sandage et al.
1998	65.0 ± 1.0	3.3	Watanabe et al.
1998	44.0 ± 4.0	6.1	Impey et al.
1999	60.0 ± 2.0	4.1	Saha et al.
1999	55.0 ± 3.0	4.4	Sandage
1999	54.0 ± 5.0	2.9	Bridle et al.
1999	42.0 ± 9.0	2.9	Collier et al.
2000	65.0 ± 1.0	3.3	Wang et al.
2000	52.0 ± 5.5	3.0	Burud et al.
2004	78.0 ± 3.0	3.2	Wucknitz et al.
2006	74.9 ± 2.3	3.0	Ngeow & Kanbur
2006	74.0 ± 2.0	2.9	Sánchez et al.
2008	61.7 ± 1.2	5.7	Leith et al.
2012	74.3 ± 2.1	2.9	Freedman et al.
2013	76.0 ± 1.9	4.1	Fiorentino et al.
2016	73.2 ± 1.7	2.9	Riess et al.
2018	73.5 ± 1.7	3.1	Riess et al.
2018	73.3 ± 1.7	3.0	Follin & Knox
2018	73.2 ± 1.7	2.9	Chen et al.
2019	74.0 ± 1.4	4.1	Riess et al.

the values at Table 1, we see many points later than the year 2000 in the list of outliers, which have $> 2.8\sigma$ deviation. Certainly, the absolute values of H_0 measurement after the year 2000 are closer to the value of the weighted average, because the error bars are also smaller than those ones before 2000. However, the important thing here is not whether the error bars are lower, but whether the error bars are accurately measured, and we have a bunch of values measured after 2000 that are not within this category, assuming that $\overline{H_0} = 68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ represents the real value.

We may think that the value of $\overline{H_0} = 68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is not correct, since it was obtained with the weighted average of the whole sample 1976-2019. We repeat the whole analysis only with the subsample of 2001-2019 data: they are 85 measurements. For this, we obtain $\overline{H_0} = 69.44 \pm 0.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\chi^2 = 195.2$. The probability that the observed distribution is due to chance is $Q = 7.7 \times 10^{-11}$. In order to reach a value for Q that is statistically significant ($Q \geq 0.05$), at least six outliers (with more than 2.8σ deviation from the average value) must be removed from the data ($n = 79$, $\chi^2 = 94.8$), producing a value for Q of 0.095. Table 2 lists the six measurements with more than 2.8σ deviation. The conclusion is that the data of 2001-2019 are significantly better (only 7% of data need to be rejected to get a probability $Q > 0.05$), but there are also tensions here. This includes the recent tension of CMBR vs. supernovae data given from Riess et al. (2019) within Λ CDM cosmology, but there are also other measurements with unacceptable high deviations from the average. The fact there is less relative deviation in more recent data might be due to the non-independence of the data; if we took truly independent measurements, using different techniques, this ratio would presumably be much higher.

Table 2. Outliers: measurements of H_0 in which $|H_0 - \overline{H_0}| > 2.8\sigma$, where $\overline{H_0} = 69.44 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the weighted average of the 85 values of the literature between (included) 2001 and 2019.

Year	H_0 (km s ⁻¹ Mpc ⁻¹)	$\frac{ H_0 - \overline{H_0} }{\sigma}$	Authors
2002	44.0 ± 9.0	2.8	Winn et al.
2004	78.0 ± 3.0	2.9	Wucknitz et al.
2008	61.7 ± 1.2	6.7	Leith et al.
2013	76.0 ± 1.9	3.5	Fiorentino et al.
2018	67.4 ± 0.5	4.1	Chen et al.
2019	74.0 ± 1.4	3.2	Riess et al.

We might think that the problem of a too low Q in this last analysis with the subsample 2001-2019 can be avoided if we remove some points we suspect to be wrong, for instance the value of $61.7 \pm 1.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ given by Leith, Ng & Wiltshire (2008) using SNe Ia. However, removing some values because we do not like them and do not agree the dominant trends is not a valid objective method of data analysis. We can see that there were important Hubble tensions long before the announcement of Riess et al. and a rigorous statistical analysis cannot decide that some values are more trustable than others. Leith, Ng & Wiltshire (2008) might have underestimated their errors, and Riess et al. (2019) might also have underestimated the errors. Saying that we trust one team more than another is not a scientific approach.

As said in the previous subsection, changing the weighted average ($69.44 \pm 0.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ herein) for another value does not solve anything, but makes the statistics worse. For instance, if we choose as reference the value of $74.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ given by Riess et al. (2019), we obtain a $\chi^2 = 561.9$ for the 85 points (85 degrees of freedom), with associated $Q = 2.4 \times 10^{-71}$, and in order to reach a $Q \geq 0.05$, we would have to remove at least eight measurements.

3 RECALIBRATION OF PROBABILITIES

In Fig. 1, we plot the frequency of deviation larger than $x\sigma$ from the weighted average value $H_{0,\text{teor.}} = 68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ derived using the whole sample of 163 measurements (including the outliers). Clearly, the probabilities are much higher than those expected in a normal Gaussian error distribution. For example, in a Gaussian error distribution we should get a $P = 2.7 \times 10^{-3}$ of obtaining a deviation higher than 3σ (where σ is the error of the measurement), but instead we observe that 11.7% of our measurements get deviations higher than 3σ . The fit of our probability with the frequencies we obtain from the real measurements is:

$$P(|H_0 - \overline{H_0}| > x\sigma) = (0.93 \pm 0.06) \times \exp[-(0.720 \pm 0.013)x]. \quad (2)$$

This is equivalent (fit in the range of x between 1 and 12) to a number of σ s deviation in a normal distribution (see Fig. 2):

$$x_{\text{eq.}} = (0.830 \pm 0.004)x^{0.621 \pm 0.003}, \quad (3)$$

where x is the number of σ s in the measurement (i.e. $x = \frac{|H_0 - \overline{H_0}|}{\sigma}$). Hence, for instance, a datum that is 3.0σ away from the expected value should not be interpreted as a 3.0σ tension, but a 1.6σ one in equivalent terms of a normal distribution ($P(> x_{\text{eq.}}) = 0.11$). Likewise, a tension of 4.4σ (as for instance claimed by Riess et al. (2019)) for a Hubble tension) is indeed a 2.1σ tension in equivalent terms of a normal distribution, with an associated $P(> x_{\text{eq.}}) = 0.036$ (1 in 28), which is not large but it can occur as a random statistical

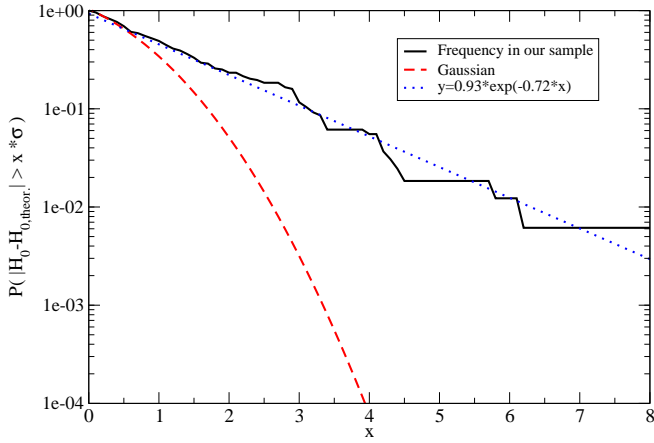


Figure 1. Probability of deviation larger than x sigmas, assuming $H_{0,\text{teor.}} = 68.26 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (the weighted average value). The dotted line shows the best exponential fit. The dashed line denotes the expected probability if the

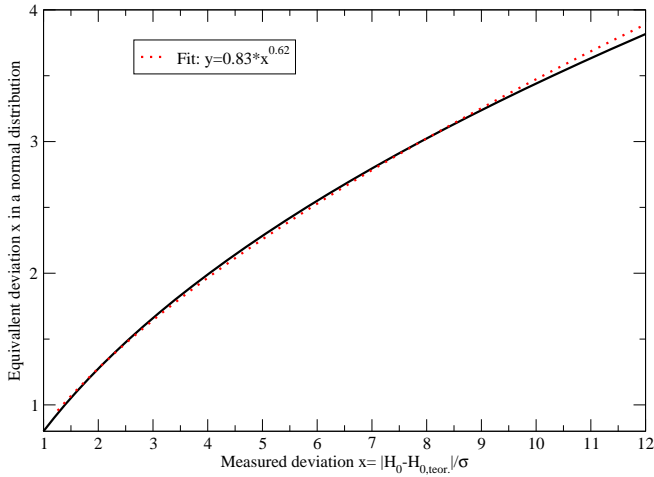


Figure 2. Equivalent number of σ of a normal distribution as a function of the number of measured σ of deviation with respect the weighted average value of H_0 , assuming that the probabilities are given by the blue dotted line of Fig. 1 (Eq. 2).

fluctuation. For an even larger limit of the tension, at 6σ , as pointed by using some different datasets (Di Valentino, Mena & Pan 2021), we would have a equivalent 2.5σ with an associate $P(> x_{\text{eq.}}) = 0.012$ (1 in 83), which is still not amazing.

4 CONCLUSIONS

We have examined the trend and dispersion of values of the measurements of H_0 during 43 years (1976–2019). The probabilities Q for the distribution of different measurements of H_0 and their errors are extremely low, both with respect to a constant value (weighted average of all the measurements) or with a linear fit (Faerber & López-Corredoira 2020). We need removing 24–27 measurements to reach a statistically significant dataset ($Q \geq 0.05$).

In the light of the analysis carried out here, one would not be surprised to find cases like the 4.4σ discrepancy seen between the best measurement using Supernovae Ia in Riess et al. (2019). It is likely that the underestimation of error bars for H_0 in many measurements

contributes to the apparent 4.4σ discrepancy. Here we have carried out a recalibration of the probabilities with the present sample of measurements and we find that $x\sigma$ deviation is indeed equivalent in a normal distribution to the $x_{\text{eq.}}$ σ deviation, where $x_{\text{eq.}} = 0.83x^{0.62}$. Hence, the tension of 4.4σ , estimated between the local Cepheid–supernova distance ladder and cosmic microwave background (CMB) data, is indeed a 2.1σ tension in equivalent terms of a normal distribution, with an associated $P(> x_{\text{eq.}}) = 0.036$ (1 in 28). This is not large but it can occur as a random statistical fluctuation. This can be increased up to a equivalent tension of 2.5σ in the worst of the cases.

One may be tempted to claim that the statistics and treatment of uncertainties in published H_0 measurements from the last century cannot be applied to the interpretation of the most recent measurements, because we know the reason why the error bars measured > 20 years ago were underestimated, whereas we are totally certain that our present-day measurements of the error bar are accurate. However, this statement would be a misunderstanding of the present analysis: we are not here discussing the physics and assumptions (as discussed in the introduction) behind those measurements, and certainly we do not have any argument to criticize the techniques of recent H_0 measurements. Rather, we are doing here a metaanalysis from a historical point of view. Of course, Riess et al. think that their error bars are accurate, but can we be sure that within 30–40 years the same security can be maintained? We can also assume that Sandage, Tammann and other very high-qualified specialists measuring H_0 in the 1970s were also convinced that their measurements were correctly carried out, but it was later proven that they have underestimated their errors. The idea that scientists in the past were not accurate enough in their statistical analyses and present-day researchers are doing a perfect job is not justified from an objective historical point of view. In this sense, this investigation makes sense. The question is: how frequent was to get an underestimation of the error bars in H_0 between 1976 and 2019? And we got an answer, which is useful to understand present and future data analyses.

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DATA AVAILABILITY: The data underlying this article are available in the article and the cited references, and in its online supplementary material.

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